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USING A SIMPLIFIED COMPUTER PROGRAM
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DESIGN AND PERFORMANCE ANALYSIS OF
SOLID-PROPELLANT ROCKET MOTORS
USING A SIMPLIFIED COMPUTER PROGRAM

By Richard H. Sforzini

Auburn University
Auburn, Alabama

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Final Report



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16. ABSTRACT This report presents an analysis and a computer program which represents a compromise between the more sophisticated programs using precise burning geometric relations and the textbook-type of solutions. The program requires approximately 900 computer cards including a set of 20 input data cards required for a typical problem. The computer operating time for a single configuration is approximately 1 minute and 30 seconds on the IBM 360 computer. About 1 minute and 15 seconds of the time is compilation time so that additional configurations input at the same time require approximately 15 seconds each. The program uses approximately 11,000 words on the IBM 360. The program is written in FORTRAN IV and is readily adaptable for use on a number of different computers; e.g., IBM 7044, IBM 7094 and Univac 1108.					
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ERRATUM

All of the computations in the main report and in the appendix were performed with the following error in the programming of the Newton-Raphson solution of the burning rate equation: PONOZ was used in place of PNOZ in Table IV-2, p. 43, line 16 of the report. The program statement is correct in the report, but because of the error numerical results obtained from the program will differ slightly from those presented in the report especially during the initial portion of the traces. The variations are not large enough to change the conclusions of the report or to invalidate the use of the design charts presented in the appendix.

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NOMENCLATURE

An asterisk before the symbols indicates an input variable. All input subscripted and non-subscripted variables are listed separately.

<u>English Symbol</u>	<u>Definition</u>	<u>Units Used</u>
*a	Propellant burning rate coefficient	in/sec-psi ⁿ
A	Cross-sectional or flow area	in ²
A _{bm}	Main burning surface evaluated during tailoff	in ²
A _{bn} , A _{bp} ,	Total burning surface associated with nozzle	in ²
*A _{bs}	end, port and slot surfaces, respectively	
*A _{bnT}	Burning surfaces (as a function of y) at nozzle end for tabular input	in ²
*A _{bpT}	Burning surfaces (as a function of y) of port sides for tabular input	in ²
*A _{bsT}	Burning surfaces (as a function of y) of slots for tabular input	in ²
*A _{bto}	Sliver burning surface evaluated during tailoff	in ²
A _{Gs}	Initial burning area of star grain flat ends	in ²
A _p	Port area	in ²
*A _{phT} , *A _{pnT}	Controlling port areas at head and aft ends, respectively (as a function of y) for purely tabular inputs	in ² in
*B _{bp} , *B _{bs}	Additive burning surfaces input as special equations for port, slot and nozzle surfaces, respectively	in ²
C	Characteristic exhaust velocity	ft/sec
C _F	Thrust coefficient	_____

NOMENCLATURE (Continued)

<u>English Symbol</u>	<u>Definition</u>	<u>Units Used</u>
D	Diameter	in.
D*,*D _i *	Instantaneous and initial diameters, respectively, of the nozzle throat	in.
*D _{cc}	Estimated mean diameter of the cylindrical portion of the case	in.
*D _e	Diameter of the nozzle exit	in.
*D _i	Length average initial diameter of controlling length of circular perforated grain	in.
*D _o	Length average outside diameter of circular perforated grain, excluding lengths in closures	in.
D _{TP}	Initial diameter of the thrust termination passageways in the grain	in.
*r	Fillet radius at star valley	in.
*E _n	Radial erosion rate of the nozzle throat	in/sec
F	Thrust	lbf.
g _o	Gravitational constant	ft/sec ²
*G _{op}	Integer designating type of grain perforation	_____
h	Instantaneous vehicle altitude	ft.
*h _b	Estimated burnout altitude	ft.
*h _{cn}	Axial length of nozzle closure	in.
*I _{op}	Integer designating type of program input	_____
I _{sp} ,I _T	Specific and total impulse, respectively	lbf-sec/lbm,lbf-sec
J	Throat-to-port cross-sectional area ratio	_____

NOMENCLATURE (Continued)

<u>English Symbol</u>	<u>Definition</u>	<u>Units Used</u>
$*K_1, *K_2$	Empirical constants in nozzle weight estimation equation (Eq. 63)	lbm/in ³ , in/sec
$*K_{eh}, K_{en}$	Erosion rate of insulation at head and nozzle end of case, respectively	in/sec
l_f	Distance between center line of motor and fillet center of standard star grain	in.
l_s	Length of sides of truncated star point excluding fillet	in.
$*l_{TP}$	Initial length of termination passages between centers of gravity of perimeters of bases	in.
$*L$	Total initial perforated grain length including gaps at slots	in.
$*L_{cc}$	Length of cylindrical portion of case including forward and aft segments	in.
L_{Gc}, L_{Gci}^*	Instantaneous and initial total axial lengths, respectively, of circular perforated grain (not including gaps)	in.
$*L_{Gni}$	Initial axial length	in.
$*L_{Gsi}$	Initial total length of star-shaped perforated grain	in.
$*L_{Ta}$	Estimated length of grain at the aft end at start of first tailoff having an additional taper not represented by z_o or θ_G	in.
m	Mass	slugs
M	Mach number	_____
$*n$	Burning rate exponent	_____
$*n_1, *n_2$	Number of standard derivations in maximum chamber pressure and case yield strength, respectively, to be used as bases for design	_____

NOMENCLATURE (Continued)

<u>English Symbol</u>	<u>Definition</u>	<u>Units Used</u>
*n _n	Number of burning flat end surfaces of a star grain located at the extreme nozzle end of the chamber	_____
*n _p	Number of star points	_____
*n _s	Number of burning flat end surfaces of a star grain not located at the nozzle end of the chamber	_____
n _{seg}	Number of case segments	_____
*n _T	Number of thrust termination ports	_____
P	Pressure	lbf/in ²
*Q _{op}	Integer designating grain arrangement	_____
r	Burning rate	in/sec
R	Radius	in.
*R _c	Outside radius of a star grain	in.
R _{eL}	Initial Reynolds number of gases in propellant port based on port length	_____
*R _p	Initial radius of truncated star points	in.
*S	Number of burning flat slot sides of a circular perforated grain not including the nozzle end	_____
S _G	Burning perimeter of a star grain	in.
*S _{op}	Integer designating type of star grain	_____
S _y	Yield strength	lbf/in
*S _{ynom}	Nominal yield strengths of the case and nozzle structural materials, respectively	lbf/in
*S _{yn}		

NOMENCLATURE (Continued)

<u>English Symbol</u>	<u>Definition</u>	<u>Units Used</u>
t	Time	sec.
t_b	Calculated total operating time of motor	sec.
$*t_{bl}$	Estimated time at burnout	sec.
t_t	Time at which propellant burning ceases	sec.
T_l	Loss in length of burning propellant associated with additional tailoff controlling taper near the nozzle end of the grain	in.
u	Velocity	ft/sec
$V_c, *V_{ci},$ $*V_{cf}$	Instantaneous, initial and final volume, respectively, of chamber gases	in.
W	Weight	lbm.
$W_{p1}, W_{p2},$ W_p	Weight of propellant burned based on mass discharge rate, volume calculations, and the arithmetic average of W_{p1} and W_{p2}	lbm.
x_{out}	Distance burned at which propellant breaks up	in.
$*x_{Ta}$	Difference in web thicknesses at ends of L_{Ta}	in.
X	Ratio of the sum of the grain burning areas excluding nozzle end burning areas to the total sum of the burning areas	—
y	Distance propellant has burned from initial surface	in.
$*z_o, z$	Initial and instantaneous differences, respectively, between web thickness at head and nozzle ends of controlling grain length, excluding any initial length associated with L_{Ta} or θ_G	in.

NOMENCLATURE (Continued)

<u>Greek Symbol</u>	<u>Definition</u>	<u>Units Used</u>
$*\alpha_{eb}$	Erosive burning coefficient in the Robillard-Lenoir rule	$\text{in}^{2.8}\text{-ft}^{0.8}/\text{sec}^{1.8}\text{lb}^0.8$
$*\alpha_n$	Nozzle exit half angle	radians
$*\beta$	Erosive burning pressure coefficient in the Robillard-Lenoir rule	_____
$*\gamma$	Specific heat ratio of propellant gases	_____
Γ	A function of γ given by Eq. 4	_____
$*\delta_c, * \delta_l, * \delta_{ins}$	Specific weights of case, liner and case insulation, respectively	lbm/in^3
ΔA_{bT}	Burning surface area change resulting from thrust termination passageways	in^2
$*\Delta D_i$	Difference between initial circular perforated grain diameter at nozzle end of controlling length and D_i	in.
$\Delta P_c / \Delta y$	Rate of change of chamber pressure with respect to distance burned	lb/in^3
$*(\Delta P / \Delta y)_{out}$	Depressurization rate at which propellant is extinguished	lb/in^3
ΔT_i	Maximum expected increase in initial grain temperature above design point condition	$^{\circ}\text{F}$
$\Delta y, \Delta y_1$	Incremental distance burned and initial value of same	in.
ϵ	Nozzle expansion ratio	_____
ζ_F	Thrust loss coefficient	_____
ζ_M	Motor mass fraction; i.e., W_p / W_M	_____
$*\theta_{cn}, * \theta_{ch}$	Approximate acute angle bonded circular perforated grain makes with motor center line at the head and nozzle closures, respectively. Also referred to as the grain contraction angles.	radians

NOMENCLATURE (Continued)

<u>Greek Symbol</u>	<u>Definition</u>	<u>Units Used</u>
$*\theta_f$	Angular location of fillet center of standard star from line of symmetry	radians
$*\theta_G$	Angle burning surface element of circular grain makes with longitudinal axis of motor at the nozzle end of the chamber at the nozzle	radians
$*\theta_n$	Nozzle cant angle	radians
$*\theta_p$	Angle of standard star point	radians
θ_s	Half angle of star grain sector	radians
θ_{sl}	Angular location of slot side of truncated star grain	radians
$*\theta_{TP}$	Acute angle between axis of thrust termination passage and motor axis	radians
$*\mu$	Absolute viscosity of chamber gases	lbf-sec/in-ft
$*(\pi_p)_K$	Temperature sensitivity coefficient of pressure at constant $K = A_b/A^*$	/°F
$*\rho_p$	Solid propellant density	slugs/in ³
$*\sigma_P/P, * \sigma_{S_{yc}}/S_{yc}$	Statistical variation in P and S_y respectively; i.e., one standard deviation/the nominal value of the variable	—
$*\tau_l$	Thickness of the liner	in.
$*\tau_s$	Thickness of propellant web at slot bottom of truncated star grain	in.
$*\tau_{Teff}$	Estimated "effective" web thickness of termination port	in.
$*\tau_w$	Web thickness of main propellant grain	in.
$*\tau_{ws}$	Web thickness of standard star (same as τ_w except for some combination grains)	in.

NOMENCLATURE (Continued)

<u>Greek Symbol</u>	<u>Definition</u>	<u>Units Used</u>
$*\psi_a$	Safety factor on nozzle ablative material	_____
$*\psi_c$	Safety factor on case thickness	_____
$*\psi_{ins}$	Safety factor on nozzle insulation thickness	_____
$*\psi_s$	Safety factor on nozzle structural material	_____

Subscripts

a	Additional or ambient
av	Arithmetic averages between head and nozzle end values
c	Case, chamber value or cylinder
d	Dome
D	Discharged
e	Exit or erosion
f	Final
G	Generated
h	Head end of grain
i	Initial
ins	Insulation
j	Incremental index; i.e., number of increments burned
l	Liner
M	Motor
max	Maximum value

NOMENCLATURE (Concluded)

Subscripts (Cont'd)

meop	Maximum expected operating condition
n	Nozzle or nozzle end of grain
nc	Nozzle end of circular perforated grain
ns	Nozzle end of star grain
nT	Nozzle end tabular value
nom	Nominal value
o	Stagnation
pc	Port of circular perforated grain
ps	Port of star grain
pT	Port tabular value
sc	Slot of circular perforated grain
ss	Slot of star grain
sT	Slot tabular value
vac	Vacuum
w	Value at web time

Superscripts

*	Choked throat value
-	Arithmetic average value over Δy
.	Time rate of change

I. INTRODUCTION AND SUMMARY

This report presents the results of research performed at Auburn University during the period from June 16 to October 12, 1972, under Modification No. 6 to the Cooperative Agreement, dated October 6, 1969, between the NASA Marshall Space Flight Center and Auburn University. The principal objective of the research was to develop a simplified computer program which would serve as an aid in preliminary design of solid-propellant rocket motors (SRMs). The program would also permit generation of SRM parametric and performance data which were to be compiled as a part of the task.

A number of computer programs are already available to government and industry for use in SRM design which will give results based on quite precise mathematical formulation of burning surface geometry. The general tendency has been toward developing more and more sophistication in these programs. This creates substantial difficulty for the user in understanding the program and preparing inputs; lengthy orientation times are frequently required. Also use of such programs involves long operating and turn-around times on the computer and attendant high costs. Elementary textbook-type approaches are of course possible, but the accuracy requirements must be considered. Indeed, in view of the high cost associated with SRM development, a propulsion contractor would be ill-advised in not making some use of a tried and proven sophisticated program available to him. On the other hand, it is questionable whether the accuracy obtained with the sophisticated programs is sufficient to justify their adoption for the bulk of preliminary design work. Unknowns such as the erosive burning characteristics of a new propellant, or even a new SRM design with a well characterized propellant, cast doubt on the accuracy of performance predictions. In view of this, a less precise approach is often acceptable in preliminary design; for example, in determining the effects to anticipate from a given design change.

This report presents an analysis and a computer program which represents a compromise between the more sophisticated programs using precise burning geometric relations and the textbook-type of solutions. The program requires approximately 900 computer cards including a set of 20 input data cards required for a typical problem. The computer operating time for a single configuration is approximately 1 minute and 30 seconds on the IBM 360 computer. About 1 minute and 15 seconds of the time is compilation time so that additional configurations input at the same time require approximately 15 seconds each. The program uses approximately 11,000 words on the IBM 360. The program is written in FORTRAN IV and is readily adaptable for use on a number of different computers; e.g., IBM 7044, IBM 7094 and Univac 1108.

Based on input of solid-propellant rocket initial geometric properties, data on the characteristics of the propellant and inert materials, and design criteria constants, the program computes the weights of major components and

performance characteristics of the SRM including specific impulse, thrust, chamber pressures, and other significant time varying parameters. The program may be used for rocket motors of all sizes; however, primary consideration was given in this study to very large SRMs (120 in. dia. and above). Therefore, some of the empirical expressions for component weights developed, such as the one for nozzle weights, may be more accurate for larger motors than for smaller ones as discussed further in the body of the report.

With large motors again in mind, particularly those under consideration for application to the Space Shuttle Booster (References 1-4), the program was designed to treat segmented configurations with both star and circular perforated grains present in various arrangements in the same SRM. In addition, monolithic grains and either standard star or truncated star (slotted tube) shapes may be analyzed. While there is considerable flexibility provided in specifying the geometry of the ends of circular perforated grains, there are severe constraints on the specification of the geometry of star grains and on passageways in the grain for thrust termination ports. To offset this, as a program option, the user may specify tabular data to correct precisely or approximately for variation in burning surface geometry with distance burned. Alternately, the entire burning area geometry may be specified by tabular input.

The key to the relative simplicity of the program, aside from the use of end constraints on burning surface geometry, is the way in which the variation in surface regression along the length of the grain is treated. The more sophisticated programs result from mathematically dividing the grain axially into a large number of segments. For each segment, in turn, the governing fluid dynamic equations are solved. The difficulty here is that the flow velocity at each segment is unknown until the burning rate is determined, but the burning rate depends on the velocity. Therefore, an iterative technique must be used which requires assumption of the conditions at the head end of the grain, solution of the governing equations for each segment in turn and finally a comparison of the total amount of mass generated with that theoretically discharged through the nozzle. If the check fails, a new assumption and repetition is required to obtain a solution for one instant of time. The advantage of this method is that it permits the surface regression to be determined at each segment for each instant of time, but it is clear that this results in both long computation time and large computer storage capacity. The present analysis avoids the above mentioned difficulties through the assumption that the surface regresses in a linear fashion along the length of grain consistent with the difference in surface regression at the head and aft ends for which regression (burning) rates are separately calculated. Support for this assumption as an approximation may be found in the results of interrupted burning tests (See for example Reference 5, p. 418). As already noted, the nature of erosive burning characteristics is at best poorly known which casts some doubt on a need for a more rigorous model of the burning surface geometry. Separate determination of head and

nozzle end burning rates taking into account erosive burning effects provides a means of accounting for one of the principal effects of a surface regression which varies along the length of the grain. This is the effect on tailoff characteristics which, in somewhat simplified terms, is treated in the present analysis by reducing the burning surface at tailoff in proportion to the length of propellant that has experienced burnout according to the linear model of the regressing surface.

A discussion of the program input and output variables is presented in Section II of this report. This will serve as a guide to the user in proper specification of his problem and interpretation of the output. The discussion should also be read by those desiring to understand the mathematical analysis which is outlined as a step-by-step procedure in Section III. However, Section III may be omitted without loss of continuity by the user interested only in understanding how to specify his problem. Section IV presents the information required to operate the program and computer format and flowcharts. Once the input variables discussed in Section II have been specified, the information given in Section IV should suffice for anyone familiar with use of a computer having FORTRAN IV capability to obtain the program output. Also, the program listing and flowcharts in Section IV will be useful for determining how to modify the program and for a detailed understanding of the program logic. However, for those desiring to understand only the basic logic of the program, study of Section III is recommended, as it was written as a step-by-step procedure with the purpose in mind of conveying the basic program logic. Section V presents test cases and comparison with results of others, both theoretical and experimental. Again those seeking only to know how to use the program may omit this section, although the user will find the example listings of input and output data, which are presented only here, helpful. Finally, the Appendix is a design data compilation resulting from use of the program which may be of aid to the designer in predicting effects of various design parameters on SRM characteristics and performance.

II. DISCUSSION OF INPUT AND OUTPUT

The computer program described in this report is capable of calculating performance for a large variety of different grains and combinations. On the other hand, to attain the goal of a simplified program, it is necessary to impose substantial restrictions on the geometric end constraints of the burning grain surfaces. Thus the accuracy of results will depend to a large degree on judicious selection of the geometric input variables. The difficulties encountered in approximating the geometric burning characteristics may be lessened, of course, by making use of the program options which permit specification of the burning geometry of part or all of the grain by using tabular inputs with the distance burned being the single argument. In any event, it is necessary that the program user have a clear understanding of the definition of each input variable so that he can appropriately specify each configuration to be analyzed. In addition, the user must understand how to specify proper design criteria and the physical characteristics of propellant and inert materials.

In this section each input variable is defined in the order in which it is encountered in the program. The English or Greek symbol is given first followed by the computer symbol in parenthesis. Sketches of geometric characteristics are given for clarification. Where appropriate, additional discussion of the variable and recommended or typical numerical values are given. In addition, the present outputs of the program are defined. It is the aim of this section to provide a guide to the user in the specification of his problem and interpretation of the outputs.

Propellant Characteristics

ρ_p (RHO)	Density of the solid propellant (slugs/in. ³).
a (A)	Propellant burning rate coefficient in the equation $r = aP^n$ (in/sec-psia ⁿ).
n (N)	Burning rate exponent (1).
α_{eb} (ALPHA)	Erosive burning coefficient in the Robillard-Lenoir burning rate rule (Equation III-11)(in ^{2.8} -ft ^{0.8} /sec ^{1.8} lbf ^{0.8}).
β (BETA)	Erosive burning pressure coefficient in the Robillard-Lenoir rule (1).
μ (MU)	Absolute viscosity of the chamber gases (lbf-sec/in-ft).
C* (CSTAR)	Characteristic exhaust velocity (ft/sec). This is the gas dynamic characteristic velocity which can be obtained from the expression $C^* = \sqrt{RT_c}/\Gamma$ where R is the gas constant and T_c the adiabatic flame temperature of the propellant.

Primary Basic Motor Dimensions

L (L)	Total initial perforated grain length including gaps at slots (in.). This is used only for calculation of the initial length Reynolds number and in the erosive burning rate equation.
τ_w (TAU)	Web thickness of the controlling propellant length (in.). (See ZO below for definition of controlling length). If the grain is tapered, the length average value should be specified excluding lengths having additional taper not represented by ZO and segments located anywhere having a step decrease in thickness. Such step decreases must be handled by the additive tabular input option if they introduce two significantly different web thicknesses for the same grain type; e.g., two circular perforated grains. If a circular perforated grain is used, it is assumed that it will have the approximate average web thickness of the controlling propellant length.
V_{ci} (VCI)	Initial volume of chamber gases (in ³). This is the volume of the empty case less the volume of the propellant. A reasonable estimate based on the case volume and the estimated volumetric loading density will suffice.
V_{cf} (VCF)	Final volume of chamber gases (in ³). This is the volume of the empty case. A reasonable estimate will suffice.
D_i^* (DTI)	Initial diameter of the nozzle throat (in.).
α_n (ALFAN)	Exit half angle of the nozzle (radians).
θ_n (THETA)	Cant angle of the nozzle with respect to the motor axis (radians).
L_{Ta} (LTAP)	Estimated length of grain at the aft end at start of first tailoff having an additional taper not represented by ZO or THETAG (in.). See Figure II-1. This variable permits the designer to specify an additional taper at the nozzle end of a circular perforated or star grain to produce regressivity shortly before tailoff.
x_{Ta} (XT)	Difference in web thickness at ends of LTAP (in.). See Figure II-1.
z_o (ZO)	Initial difference between web thickness at the head and aft ends of controlling grain length, not including any initial length associated with LTAP or THETAG (in.). The controlling length of the grain is the axial length between the head end of the grain and the position of expected maximum Mach number in the port. It may be assumed, in most cases, that this length terminates whenever there is an abrupt decrease in web thickness near the aft end of the chamber or when θ_G is greater than 5°.

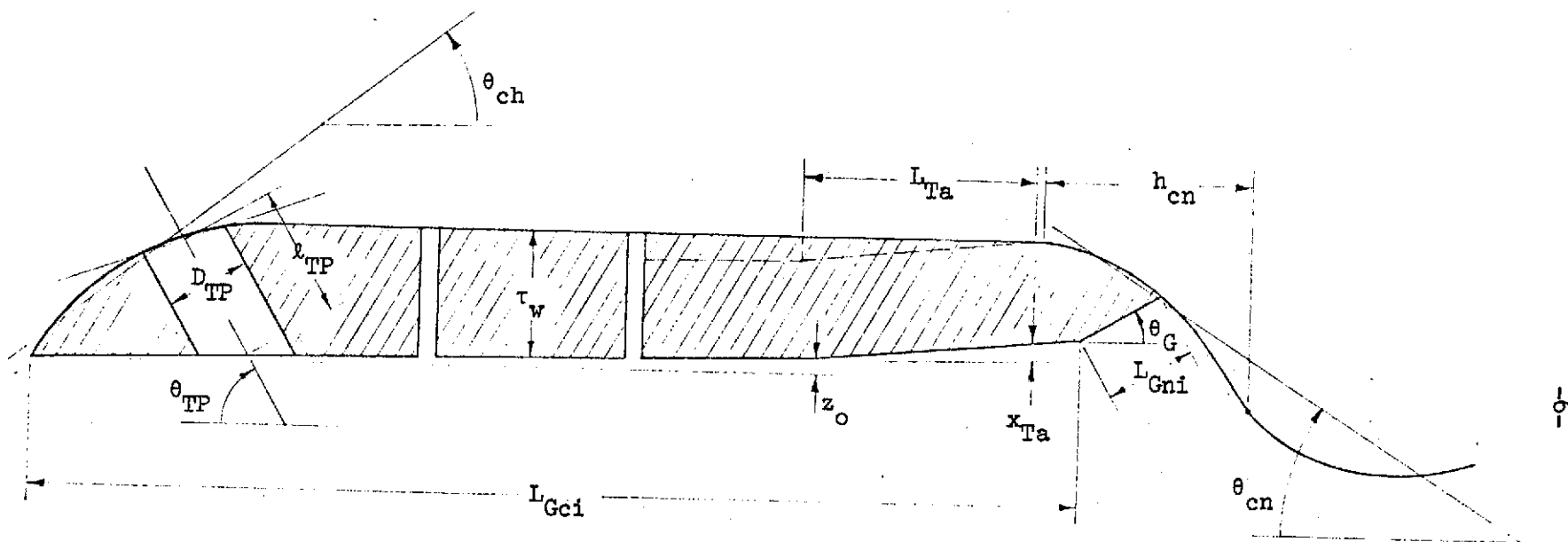


Figure II-1. Basic motor dimensions. L_{Gci} , L_{Gni} , and θ_G are used only for circular perforated grains. The controlling grain length may be L_{Gci} or some lesser value depending on the position of maximum port Mach number. τ_w is the length average web thickness over the grain length excluding the lengths associated with L_{Ta} , θ_G and the head end dome.

Basic Performance Constants

Δy_1 (DELTAY)	Initial desired burn increment (in.). This increment will be used by the program for the initial 5% of the web thickness burned, for the period shortly preceding and following tailoff and at such other times as the rate of pressure change is large. Larger increments will automatically be used at other times. An initial increment size of approximately $0.005r_w$ is recommended but larger increments may be used for relatively flat burning surface versus distance burned traces.
x_{out} (XOUT)	Distance burned at which the propellant breaks up (in.). This permits the option of specifying termination of burning resulting from possible structural breakup of the propellant. If this option is not desired, XOUT should be set to some large value; e.g., 1000 in.
$(\Delta P/\Delta y)_{out}$ (DPOUT)	Rate of change of pressure with respect to distance burned at which the propellant is extinguished (lb/in^3). This permits the option of specifying termination of burning resulting from rapid depressurization. If this option is not desired, DPOUT should be set to some large value; e.g., 10,000 psia/in. However, if an abrupt tailoff is expected, a smaller value of DPOUT must be specified as the computer will not otherwise be able to handle the discontinuity. For large motors (120 in. dia. and up) a value of 500 psia/in is recommended and proportionately larger values for smaller motors.
ζ_F (ZETAF)	Thrust loss coefficient (1). In the absence of data to the contrary a value of 0.98 is recommended.
t_{bl} (TB)	Estimated burning time (sec). This and the variable that follows permits the program to calculate delivered specific impulse and thrust based on an assumed trajectory which was determined from analysis of typical large SRM applications.
h_b (HB)	Estimated burnout altitude (ft). To obtain sea level performance characteristics, HB should be set equal to zero.
λ (GAM)	Ratio of specific heats for propellant gases (1).
E_n (RADER)	Radial erosion rate of the nozzle (in/sec). A value of 0.0135 ips has been used by one propulsion company to predict the performance of 156 in. dia. motor with a graphite tape-phenolic impregnated throat.

Basic Properties for Weight Calculation

$(\pi_p)_k$ (PIPK)	Temperature sensitivity coefficient of pressure at constant $K = A_p/A^* (1/^\circ F)$.
ΔT_i (DTEMP)	Maximum expected increase in initial grain temperature above design point conditions ($^\circ F$).
(σ_p/P) (SIGMAP)	Statistical variation in the maximum calculated pressure(1); i.e., one standard deviation in maximum pressure divided by the calculated value of maximum pressure. The value used is typically 0.03 for an untried design. Lower values may be used if supported by data on a proven design.
$\sigma_{S_{yc}}/S_{yc}$ (SIGMAS)	Statistical variation in yield strength of the case material (1). A value of 0.02 is recommended. Alternately S_{yc} may be taken as the minimum yield strength in which case SIGMAS is 0.
n_1, n_2 (N1, N2)	Number of standard deviations in maximum chamber pressure and case yield strength, respectively, to be used as bases for design (1). A value of 3 gives statistical assurance that 99.5% of the time the calculated limits of the variable will not be exceeded in tests.
S_{ycnom} (SYCNOM)	Nominal yield strength of the case material (lbs/in ²). See also SIGMAS.
D_{cc} (DCC)	Estimated mean diameter of the motor case (in.). A rough estimation of the case and insulation thicknesses may be added to the grain diameter to obtain DCC.
ψ_c (PSIC)	Safety factor on case thickness (1). This factor is superimposed on the safety provided by use of the statistical limits on pressure and case strength.
δ_c (DELC)	Specific weight of the case material (lbs/in ³). For steel the approximate value is 0.283 lbm/in ³ .
L_{cc} (LCC)	Length of the cylindrical portion of the case including forward and aft segments (in.).
n_{seg} (NSEG)	The number of case segments (1). NSEG is 1 for a monolithic case.
h_{cn} (HCN)	Axial length of the nozzle closure (in.). See Figure II-1.

S_{yn} (SYNNOM)	Nominal yield strength of the nozzle structural material (lb/in ²).
ψ_s, ψ_a (PSIS, PSIA)	Safety factor on the nozzle structural and ablative materials, respectively (1). No additional allowances are made in the computation for statistical variation, so PSIS and PSIA provide the only safety factors. Values of 1.4 and 2.0 are recommended for PSIS and PSIA, respectively.
K_1, K_2 (K1, K2)	Empirical constants in the nozzle weight estimation equation (Eq. 63b). (lbm/in ³ , in/sec). For single, submerged conical nozzles using ablative exit cones with the steel body extending over about one-third of the exit cone, the values recommended based on empirical analysis of large (120 to 156 in. dia.) SRMs are $K_1 = 0.208$ lbm/in ³ and $K_2 = 0.0925$ in/sec. These values should also yield approximations of the weights for smaller and larger nozzles of the same type. For better results with larger or small motors or nozzles of different types, the K's should be redetermined.
ψ_{ins} (PSIINS)	Safety factor on the nozzle insulation (1). No additional allowance is made in the computations for statistical variation. A value of 2.0 has been used extensively in man-rated applications.
δ_{ins} (DELINS)	Specific weight of the case insulation (lbm/in ³). DELINS is typically 0.0462 lbm/in ³ .
K_{eh} (KEH)	Erosion rate of the exposed insulation taken everywhere the same except at the nozzle closure (in/sec). KEH is typically 0.003 in/sec.
K_{en} (KEN)	Erosion rate of the exposed insulation in the nozzle closure. KEN is typically 0.012 in/sec.
δ_l (DLINER)	Specific weight of the liner (lbm/in ³). DLINER is typically 0.033 lbm/in ³ .
τ_l (TAUL)	Thickness of the liner (in.). TAUL is typically 0.065 in.
W_a (WA)	Any additional weight not considered elsewhere in weight calculations (lbm). This includes weights associated with thrust termination and thrust reversal, thrust vectoring devices, thrust skirts (except for "y" joints), ignitors, instrumentation and final thermal protection of the motor case after burnout (see Eq. 64, Section III).

Input to Establish the Program and Basic Grain Configuration and Arrangement

- I_{op} (INPUT) 1 for only tabular input.
 2 for only equation input.
 3 for a combination of 1 and 2.
- G_{op} (GRAIN) 1 for an entirely circular perforated grain.
 2 for star grain only.
 3 for combination of 1 and 2.
- S_{op} (STAR) 0 for an entirely circular perforated grain.
 1 for standard star (See Fig. II-2).
 2 for truncated star (See Fig. II-3).

 A standard star may not be combined with a truncated star.
- n_T (NT) Number of thrust termination passageways in grain (1)
 NT is zero if there are no thrust termination passageways.
- Q_{op} (ORDER) 1 if a star grain is at head end and a circular perforated grain at aft controlling end.

 2 if a circular perforated grain is at both ends. A star grain segment may still be present.

 3 if a circular perforated grain is at head end and a star at the aft controlling end.

 4 if a star grain is at both ends.

 If grain = 1, value of order must be 2
 If grain = 2, value of order must be 4.

It is important to realize that ORDER establishes the controlling port area equations to be used. Thus if the nozzle end segment is not indeed the controlling one (the one that establishes the maximum Mach number in the port), ORDER should be specified to designate the actual controlling segment as the nozzle end segment. GRAIN, STAR, NT and ORDER are not used for INPUT = 1, but values must be assigned for continuity of computer operations.

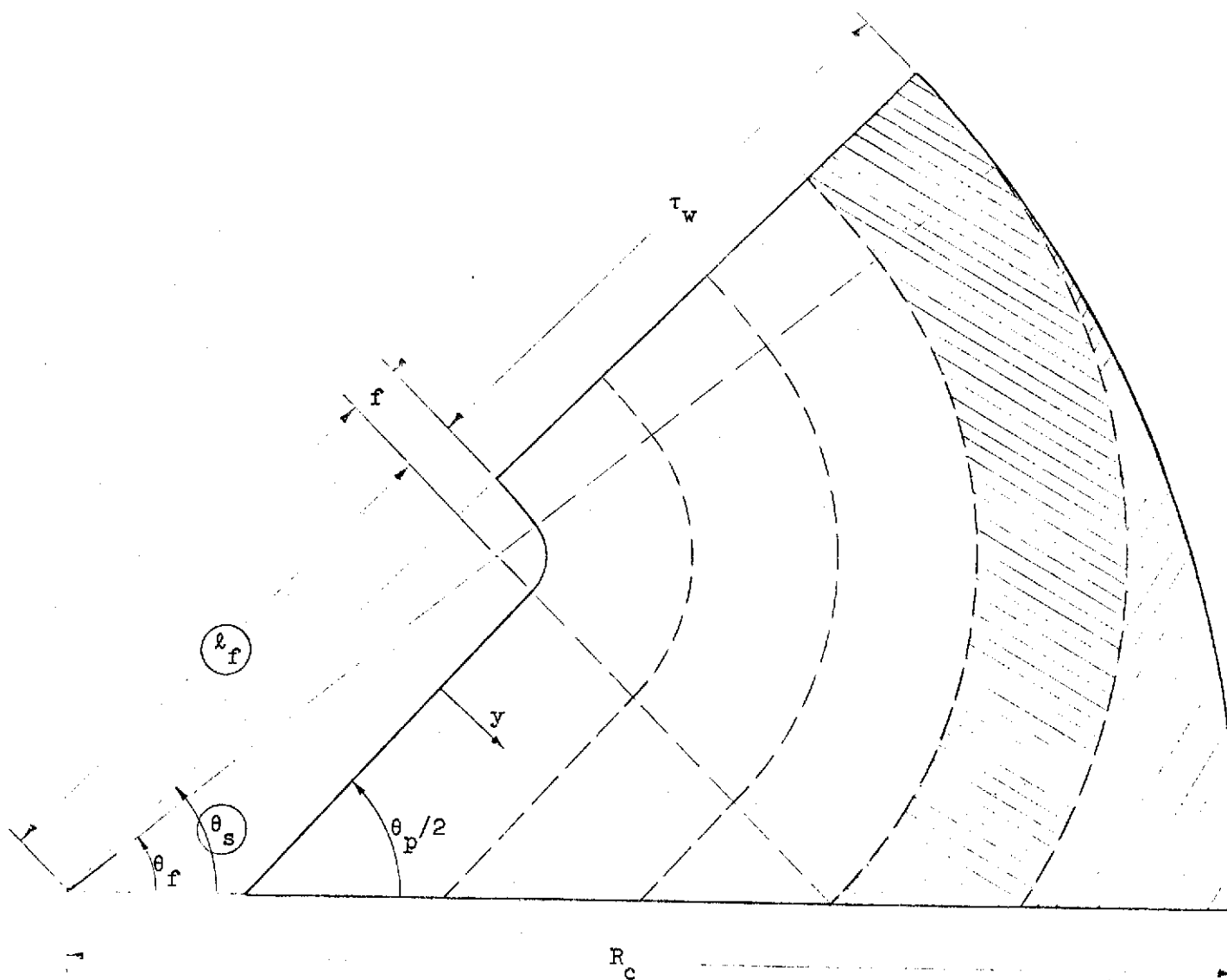


Figure II-2. Standard star grain cross-section. The evolution of the burning perimeter is shown. Calculated variables are circled. Second and third (final) zones of burning are cross-hatched.

Tabular Burning Surface and Port Areas (Not required for INPUT = 2)

A_{bpT} (ABPK)	Burning area in the port (in ²).
A_{bsT} (ABSK)	Burning area in the slots (in ²).
A_{bnT} (ABNK)	Burning area at the nozzle end (in ²).
A_{phT} (APHK)	Port area at the nozzle end of the controlling grain length (in ²).
A_{pnT} (APNK)	Port area at the head end of the grain (in ²). APHK and APNK are not required when INPUT is 2 or 3; the equation inputs must be used to provide the information in these cases. Values of all A's used must be specified to completely describe the burning surface and port areas versus distance burned. The program computes intermediate values by linear interpolation. The number of values required is arbitrary and limited only by the storage capacity of the computer, but values must be specified for $y = 0$. Also, as an example of the procedure for specifying terminal values which must be followed, burning surfaces must be specified as zero at burnout of the tabular surfaces and at a y exceeding the highest anticipated calculated performance value. Separate input cards must be prepared for each value of y and arranged as described in Section IV of this report. The use of these tabular values in conjunction with the equation inputs (INPUT = 3) increases the flexibility of the program considerably. Frequently it is easy to estimate burning surface effects which end constraints on the equation inputs neglect. In this case a table of input values can be readily prepared from estimates of the effects. Also, the program can be run first without the tabular values and outputs used as an aid in obtaining the estimate. For example, the burning perimeters of a star grain can be determined in this way and the values used to estimate the effects of a head end closure on the star grain.

Geometry for Circular Perforated Grain - Not required for completely star-shaped grain (GRAIN = 2) or for only tabular inputs (INPUT = 1)

D_0 (DO)	Length average outside diameter of circular perforated grain, excluding lengths extending into the closures (in.).
D_i (DI)	Length average inside initial diameter of circular perforated grain (in.). Only the controlling length excluding LTAP should be considered in the averaging
ΔD_i (DELDI)	Difference between the initial circular perforated grain diameter at the nozzle end of the controlling length and DI (in.).

- S (S) Number of burning flat ends of a circular perforated grain not including an extreme aft grain end (1).
- θ_G (THETAG) Angle burning surface element of circular perforated grain located at the extreme nozzle end of chamber makes with the motor axis (radians). See Figure II-1. THETAG must be set to zero if a star grain is located at the nozzle end (GRAIN = 3, ORDER = 3) or if aft end burning surfaces are represented by tabular values. THETAG is $\pi/2$ if the circular perforated grain represented by equations has a flat burning surface located at the extreme nozzle end of the chamber. If THETAG is less than or equal to 5° , a value must be assigned (zero is satisfactory), but the effect of THETAG on burning surface area is not computed (See also LGNI and LTAP). THETAG is zero if the end surface is flat and inhibited.
- L_{Gci} (LGCI) Initial total axial length of circular perforated grain represented by equation inputs not including gaps (in.). LGCI excludes lengths associated with THETAG.
- L_{Gni} (LGNI) Initial slant length of a burning conical circular perforated grain at the nozzle end (in.). LGNI is set equal to zero if THETAG is less than or equal to 5° . In this case the length otherwise associated with LGNI should be added to LGCI. If the error in burning surface area thus introduced is deemed significant, a correction may be introduced by making use of tabular inputs in combination with the equation inputs. Basic effect of small THETAG on tailoff may be accounted for by specification of LTAP. If a nozzle end burning surface is flat (THETAG = $\pi/2$), LGNI equals one half the difference between inside and outside local grain diameters.
- θ_{cn} (THETCN) The contraction angle of a circular perforated grain bonded to the nozzle closure. See Figure II-1. Use an estimated value which yields approximately the correct volume of propellant burned. If a star shaped grain is located at the extreme nozzle end of the chamber or if tabular values are used to represent downstream burning surfaces, THETCN is $\pi/2$. THETCN is also $\pi/2$ if the extreme aft end of the grain is inhibited, but only a flat ended, inhibited grain (THETAG = 0) which does not extend into the nozzle closure may be accurately represented. THETCN is zero for a burning flat end (THETAG = $\pi/2$) which does not extend into the closure.
- θ_{ch} (THETCH) The contraction angle of a circular perforated grain bonded to the head end. See Figure II-1. Use an estimated value which yields approximately the correct volume of propellant burned. THETCH is $\pi/2$ if the extreme forward end of the circular perforated grain (bonded or not) represented by equations is flat. A head end flat burning surface is treated by proper specification of S.

Basic Geometry for Star Grains - Not required for entirely circular perforated grains (GRAIN = 1) or for only tabular inputs (INPUT = 1)

- n_s (NS) Number of burning flat end surfaces of a star grain not located at extreme nozzle end of the chamber (1).
- L_{Gsi} (LGSI) Initial total axial length of star-shaped perforated grain represented by equations (in.). No provision comparable to the use of LGNI for circular perforated grains is made here to treat effects resulting from THETAG greater than 5° . Adjustments may be made, however, by use of tabular input values in conjunction with the equation inputs. Also, effects of taper, including additional small taper at the nozzle end, on tailoff may be treated by use of the variables ZO, XT and LTAP.
- n_p (NP) The number of star points (1).
- R_c (RC) The star grain outside radius (in.).
- f (FILL) The fillet radius at star valleys (in.). See Figure II-2,3.
- n_n (NN) Number of burning flat end surfaces (0 or 1) of a star grain located at the extreme nozzle end of the chamber (1).

Special Geometry for Truncated Star - Not required for standard star grains (STAR = 1)

- R_p (RP) The length average initial radius of the truncation (in.). See Figure II-3.
- τ_s (TAUS) The length average initial thickness of the propellant web at the bottom of the slots (in.).

Special Geometry for Standard Star Grain - Not required for truncated star grains (STAR = 2)

- τ_{ws} (TAUWS) The web thickness of a standard star grain (in.). See Figure II-2. If the grain is tapered, the length average value should be used. TAUWS will be the same as TAU if the entire controlling length of grain is a star grain. Otherwise TAUWS may or may not be different from TAU.
- θ_f (THETAF) Angular location of the fillet center of standard star from line of symmetry (radians).
- θ_p (THETAP) The apex angle of the star point (radians).

Geometry of Thrust Termination Passageways - Not required if NT = 0.

l_{TP} (LTP)	Initial length of the termination passageway between the centers of gravity of perimeters of the bases (in.). See Figure II-1.
D_{TP} (DTP)	Initial diameter of the termination passage (in.).
θ_{TP} (THETTP)	The acute angle between the axis of the passage and the motor axis (radians).
τ_{Teff} (TAUEFF)	Estimated effective web thickness at the termination port (in.). The user must judge the distance burned at which the effect of the termination passage on modification of the burning surface geometry ceases to be significant. In general, this should be between two-thirds and full web thickness. The equation used to account for the burning surface is based on a passageway in a circular perforated grain terminated at the case by a flat inclined plane. Thus only a rough estimate of the effect of the termination passage is provided.

Special Equation Inputs - Required only at the option of the user. May be used when INPUT = 1, 2, or 3.

B_{bp} (BBP)	Additive burning surface input as function of Y for port burning surface (in ²).
B_{bs} (BBS)	Additive burning surface input as function of Y for slot burning surface (in ²).
B_{bn} (BBN)	Additive burning surface input as function of Y for nozzle end burning surface (in ²). In order to make use of the option of specifying the B's, a minor program modification is required. The B's are all set equal to zero in the present program. If this option is to be used the program statements assigning values to the B's are easily replaced with the desired equation inputs.

Program Outputs

The variables whose values are printed by the present program are defined below. Additional variables may also be printed with minor program modification.

Outputs as functions of time

t (T)	Operating time (sec.). This is calculated from the time initial equilibrium operating pressure is attained.
y (Y)	Average distance burned (in.).
r_n (RNOZ)	Nozzle end controlling burning rate (in/sec).
r (RHEAD)	Head end burning rate (in/sec).
P_{on} (PONOZ)	Stagnation pressure at the nozzle end of the chamber (lb/in ²).
P_h (PHEAD)	Pressure at the head end of the chamber (lb/in ²).
l/J (PTAR)	Port-to-throat cross-sectional area ratio (1).
M_n (MNOZ)	Mach number at the nozzle end of the controlling grain length (1).
S_G (SG)	Average perimeter of a star-shaped grain (in.). This will be printed as zero if there is no star grain or if tabular input values are used alone.
P_a (PATM)	The atmospheric pressure at altitude (lb/in ²). This is based on the assumed trajectory and specification of HB.
$A_{bp} + A_{bs} + A_{bn}$ (SUMAB)	The total burning surface of the propellant (in ²).
C_{Fvac} (CFVAC)	The vacuum thrust coefficient (1). No adjustments are made in the calculations for nozzle divergence, cant, or other losses.
F_{vac} (FVAC)	The vacuum thrust (lbf). Adjustments are made in FVAC and F for nozzle divergence and cant, and other losses.
F (F)	The delivered thrust based on the assumed trajectory (lbf). Losses are included.

Weight and related values

W_{p1} (WP1)	Propellant weight calculated from mass discharge rates (lbm).
W_{p2} (WP2)	Propellant weight calculated from the products of burning surfaces and incremental distances burned (lbm).

$R_{h \max}$ (PHMAX)	Maximum head end chamber pressure calculated by the program (lb/in ²).
W_p (WP)	Arithmetic average of WP1 and WP2 (lbm). A check on the calculation accuracy is provided by comparison of this with WP1 or WP2.
I_{sp} (ISP)	Average delivered specific impulse (lbf-sec/lbm). Losses are included as for FVAC and F.
I_T (ITOT)	Total delivered specific impulse (lbf-sec). Losses are included.
I_{spvac} (ISPVAC)	Average vacuum specific impulse (lbf-sec/lbm). Losses are included.
P_{meop} (PMEOP)	Maximum expected operating pressure (lb/in ²). This includes allowances for statistical variations and initial grain temperature.
τ_{cc} (TAUCC)	Thickness of the cylindrical section of the case.
W_c (WC)	Total weight of the empty case (lbm).
W_n (WN)	Total weight of the nozzle (lbm).
W_{ins} (WINS)	Total weight of the case insulation (lbm).
W_M (WM)	Total weight of the loaded case including added weight, WA (lbm).
ζ_M (ZETAM)	Motor mass fraction; i.e., the ratio WP/WM (1).
I_T/W_M (RATIO)	Total delivered impulse-to-motor weight ratio (lbf-sec/lbm)

III. ANALYSIS

This section of the report presents the mathematical analysis of the SRM design problem in a step-by-step procedure. The aim is twofold: 1) to identify the fundamental physical relationships used and 2) to establish the basic mathematical logic of the computer program.

No attempt has been made to justify completely the relationships used. The theory applied involves, in the main, basic internal ballistics and trigonometry applied in a more or less conventional way. The internal ballistics analysis follows one-dimensional frictionless compressible flow theory with the flow taken as reversible in the nozzle. Equilibrium pressure is established by iteration to find the chamber pressure which balances the mass flow rate relationship

$$\dot{m}_D = \dot{m}_G - d(\rho_c V_c)/dt$$

A number of excellent references are available which explain the fundamental relations in some detail; e.g., Reference 6, Chapters 5 and 6. Although in the present analysis fewer simplifying assumptions are made in the application of the physical principles than in the example and in similar references, the reader familiar with the basic internal ballistics of SRMs should have no difficulty in following the procedure and recognizing generalizations of the elementary approaches. In the development of the weight relationships, explanation of the assumptions made are given as changes in basic design criteria, new design approaches, or advances in the state-of-the-art may dictate changes to the equations used.

Internal Ballistics

At this point it is assumed that the port areas at the head and nozzle ends and burning surfaces areas are known as functions of the distance burned. The procedure for calculating these quantities is given later in this section. The computer program uses a subroutine in calculating the areas which is called as required.

0. Set $y = 0$ and $t = 0$

1. Compute A^* and M_n in turn from:

$$a. A^* = (\pi/4)(D_i^* + 2E_n t)^2$$

$$b. M_n = (XA^*/A_{pn})\{2[1 + (\gamma-1)M_n^2/2]/(\gamma+1)\}^{(\gamma+1)/2(\gamma-1)}$$

where $X = (A_{bp} + A_{bs})/(A_{bp} + A_{bs} + A_{bn})$

2. Compute

$$u_n = \gamma C^* M_n \{ [2/(\gamma+1)]^{(\gamma+1)/(\gamma-1)} / [1 + (\gamma-1) M_n^2/2] \}^{1/2}$$

3. $P_n/P_{on} = [1 + (\gamma-1) M_n^2/2]^{-\gamma/(\gamma-1)}$

4. $\Gamma = \gamma^{1/2} [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)}$

5. $J = A^*/A_{pn}$

6. Compute first estimate of P_{on} ,

$$P_{on} = [a_p C^* (A_{bp} + A_{bs} + A_{bn}) / A^*]^{1/(1-n)} (1 + \Gamma^2 J^2/2)^{n/(1-n)}$$

7. Compute mass flow discharge rate through nozzle

$$\dot{m}_D = A^* P_{on} / C^*$$

8. $P_n = (P_n/P_{on}) P_{on}$

9. $P_h \approx P_n + 2\dot{m}_D u_n / (A_{ph} + A_{pn})$

10. Compute head end burning rate

$$r_h = a P_h^n$$

11. Compute nozzle (aft) end burning rate

$$r_n = a P_n^n + \alpha_{eb} (\dot{m}_D X / A_{pn})^{0.8/L^{0.2}} \exp(\beta r_n \rho_p A_{pn} / \dot{m}_D X)$$

The Robillard-Lenoir erosive burning rate rule is used. Reference 7 provides support for this selection, but the relationship may be changed if desired to express r_n in terms of any of the variables available at this point.

12. Compute the mass flow generation rate

$$\dot{m}_G = (\rho_p/2) [(r_h + r_n)(A_{bp} + A_{bs}) + 2a P_{on}^n A_{bn}]$$

13. If $1.001 \dot{m}_D > \dot{m}_G > 0.999 \dot{m}_D$, the solution is complete for $y=0$.

Go to step 16

14. If not, set $\dot{m}_D = \dot{m}_G$, and

15. $P_{on} = C^* \dot{m}_D / A^*$

Return to step 8

16. Compute the initial length Reynolds number

$$R_{eL} = \dot{m}_D XL / A_{pn} \mu$$

17. Set $t = 0$.

18. Solve for M_e in

$$M_e = A^* / A_e \{2[1 + (\gamma-1)M_e^2/2]/(\gamma+1)\}^{(\gamma+1)/2(\gamma-1)}$$

19. Compute

a. $P_e / P_o = [1 + (\gamma-1)M_e^2/2]^{-\gamma/(\gamma-1)}$

b. $A_e = \pi D_e^2 / 4$

20. Compute estimated altitude (Reference 8, pp. 7-13)

$$h = h_b (t/t_b)^{7/3}$$

21. Compute atmospheric pressure (Reference 8)

$$P_a = 14.696 \exp(-0.43103 \times 10^{-4} h)$$

22. Compute the thrust coefficients

a. $C_F = \Gamma \{2\gamma[1 - (P_e/P_{on})^{(\gamma-1)/\gamma}]/(\gamma-1)\}^{1/2}$
 $+ (A_e/A^*)[(P_e/P_{on}) - P_a/P_{on}]$

b. $C_{Fvac} = C_F + (A_e/A^*)(P_a/P_{on})$

23. Compute thrust

23. a. $F = \zeta_F (\cos \theta_n) P_{on} A^* \{C_F(1 + \cos \alpha_n)/2$
 $+ (1 - \cos \alpha_n)(A_e/A^*)[(P_e/P_{on}) - P_a/P_{on}]/2\}$

23. b. $F_{vac} = \zeta_F (\cos \theta_n) P_{on} A^* \{C_{Fvac}(1 + \cos \alpha_n)/2$
 $+ (1 - \cos \alpha_n)(A_e/A^*)(P_e/P_{on})/2\}$

The above thrust equations have been arranged so that the nozzle divergence loss factor $(1 + \cos \alpha_n)/2$ modifies only the momentum term of the thrust equation.

24. Compute the instantaneous specific impulses and total delivered impulses

24. a. $I_{sp} = F/\dot{m}_D g_0$

24. b. $I_{spvac} = F_{vac}/\dot{m}_D g_0$

24. c. $I_T = \sum_j \bar{F}_j \Delta t_j$

24. d. $I_{Tvac} = \sum_j \bar{F}_{vac j} \Delta t_j$

where the bars indicate here and below the arithmetic average value over Δy except at $y=0$ where the initial value is used.

25. Set $y = \Delta y_1$ and $j = 1$

Repeat steps 1 through 3 and 5

26. Compute a first estimate of new mass flow discharge rate using the value of P_{on} at the beginning of the previous increment.

$$\dot{m}_D = A^* P_{on} / C^*$$

Repeat steps 8 through 12

27. a. Compute an estimate of dP_c/dy , the rate of change of chamber pressure
 $\Delta P_c / \Delta y = (\bar{P}_{av} / \bar{r}_{av})(\Delta \bar{r}_{av} / \Delta y)$

$$+ [\bar{P}_{av} / (\bar{A}_{bp} + \bar{A}_{bs} + \bar{A}_{bn})] \Delta (\bar{A}_{bp} + \bar{A}_{bs} + \bar{A}_{bn}) / \Delta y$$

where the subscript av indicates the arithmetic averages of nozzle and head end values.

27. b. Compute the gaseous volume of the case

$$V_c = V_{ci} + \sum_j \Delta y_j (\bar{A}_{bp} + \bar{A}_{bs} + \bar{A}_{bn})_j$$

28. If $1.002 \dot{m}_D \geq \dot{m}_G - \frac{\bar{r}_{av}}{12\Gamma^2 C^*2} [V_c \Delta P_c / \Delta y + \bar{P}_{av} (\bar{A}_{bp} + \bar{A}_{bn} + \bar{A}_{bs})] \geq 0.998 \dot{m}_D$,
the solution is complete for $y = \sum_j \Delta y_j$

Go to step 31

29. If not, set

$$\dot{m}_D = \dot{m}_G - (V_c \bar{r}_{av} / 12\Gamma^2 C^*2) \Delta P_c / \Delta y, \text{ and}$$

30. $P_{onj} = \dot{m}_D C^* / A^*$

Repeat steps 8 through 12 and then 27 through 28

31. a. Compute the time increment

$$\Delta t_j = 2\Delta y_j / (\bar{r}_h + \bar{r}_n)$$

31. b. Compute the time

$$t = \sum_j \Delta t_j$$

32. Compute the difference between web thicknesses at the head and aft ends of the controlling grain length exclusive of any length near the nozzle end having an additional taper for the purpose of controlling tailoff characteristics (See discussion of τ_w , z_o , L_{Ta} and x_{Ta} , Section III).

$$z = \sum_j \Delta t_j (\bar{r}_{nj} - \bar{r}_{hj}) + z_o$$

- 33.

Repeat steps 18 through 24c.

34. Set $j = 2$

35. If $y \geq 0.05 \tau_w$ and

$$y + \Delta y_1 < (\tau_w - |z/2| - x_{Ta})_{j-1} \quad \text{and}$$

$$|V_c \bar{r}_{av} / \Gamma^2 C^{*2}) \Delta P_c / \Delta y|_{j-1} = \dot{m}_D / 100,$$

$$\text{set } \Delta y_j = 10 \Delta y_1$$

36. If not, set $\Delta y_j = \Delta y_1$

37. Not used

38. $y = \sum \Delta y_j$

39. a. Repeat steps 1 through 3 and then 26 through 33 and 35 through 38 using \bar{A}_{pn} and \bar{A}^* in 1

39. b. Set $j = 3, 4, \dots$ Repeat steps 35 through 39b for each j
until $y \geq \tau_w - |z/2|$

39. c. Set $z = z_w$ and $\Delta y = \Delta y_w$ (j is now w)

39. d. Compute the loss in length of burning propellant T_ℓ associated with additional tailoff controlling taper near the nozzle end of the grain.

(1) If $x_{Ta} = 0$, set $T_\ell = 0$

(2) If $x_{Ta} > 0$, set

$$T_\ell = (y - \tau_w + x_{Ta} + z/2)L_{Ta}/x_{Ta}$$

If $T_\ell < 0$, set $T_\ell = 0$

If $T_\ell \geq L_{Ta}$, set $T_\ell = L_{Ta}$

40. If $z_w \leq 0$ and

40. a. If $|z_w| > \Delta y_1$ and $y < \tau_w + |z_w/2|$, set

$$\begin{aligned} A_{bp} + A_{bs} + A_{bn} = A_{bto} \left[1 + \sum_{j=w}^j \Delta y_j / z_w - \Delta y_w / 2z_w \right] \\ - A_{bm} \left[\sum_{j=w}^j \Delta y_j / z_w - \Delta y_w / 2z_w \right] \end{aligned}$$

40. b. If $|z_w| \leq \Delta y_1$, or if $y \geq \tau_w + |z_w/2|$, set

$$A_{bp} + A_{bs} + A_{bn} = A_{bm}$$

41. If $z_w > 0$ and

41. a. If $z_w > \Delta y_1$ and $y < \tau_w + z_w/2$, set

$$\begin{aligned} A_{bp} + A_{bs} + A_{bn} = A_{bm} \left[1 - \sum_{j=w}^j \Delta y_j / z_w + \Delta y_w / 2z_w \right] \\ + A_{bto} \left[\sum_{j=w}^j \Delta y_j / z_w - \Delta y_w / 2z_w \right] \end{aligned}$$

41. b. If $z_w \leq \Delta y_1$ or $y \geq \tau_w + z_w/2$, set

$$A_{bp} + A_{bs} + A_{bn} = A_{bto}$$

42. a. Compute a first estimate of P_{on} during tailoff

$$P_{on} = [a_p C^*(A_{bp} + A_{bs} + A_{bn})/A^*]^{1/(1-n)}$$

42. b. Set $P_h = P_{on}$

43. a. Compute $\dot{m}_D = A^* P_{on} / C^*$

43. b. $\Delta P_c / \Delta y = [P_{on} / (1-n) (\bar{A}_{bp} + \bar{A}_{bs} + \bar{A}_{bn})] \Delta(A_{bp} + A_{bs} + A_{bn}) / \Delta y$

43. c. $r_h = a P_{on}^n$

43. d. $r_n = r_h$

44. $\dot{m}_G = \rho_p r_{av} (A_{bp} + A_{bs} + A_{bn})$

45. Not used

46. Repeat steps 27b through 29, then 47a, b, and c below as required until 28 is satisfied

47. a. $P_{onj} = P_{on(j-1)} + (\Delta P_c / \Delta y) \Delta y_j$

47. b. $P_h = P_{on}$

47. c. Repeat steps 43b through 46

48. a. Repeat steps 31a through 33, 36 and 38

48. b. Repeat steps 40 through 47c for other values of $j > w$ until

$$P_{on} < 30 \text{ psia, or}$$

$$|\Delta P_c / \Delta y| \geq |(\Delta P / \Delta y)_{out}|, \text{ or}$$

$$y = x_{out}, \text{ or}$$

$$(A_{bp} + A_{bs} + A_{bn}) = 0$$

If any of the conditions in step 48b are met, begin "half-second trace."

49. a. Set $P_{ht} = P_h$

49. b. Set $t_t = t$

49. c. $P_n = P_{ht} \exp[-r^2 C^{*2} 12(t - t_t)/V_c]$

49. d. Set $P_{on} = P_n$

49. e. Compute $\dot{m}_D = P_{on} A^*/C^*$

49. f. Compute $t = \sum_j \Delta t_j$

49. g. Set $\Delta t_j = 0.5 \text{ sec.}$

49. h. Repeat steps 49c through 49g ten times or until

$$P_h \leq 30 \text{ psia}$$

50. Not used

51. a. Calculate weight of propellant burned

$$W_{pl} = g_o \sum_j \dot{m}_{Dj} \Delta t_j$$

51. b. $W_{p2} = \rho_p g_o \sum_j (\bar{A}_{bp} + \bar{A}_{bs} + \bar{A}_{bn})_j \Delta y_j$

51. c. $W_p = (W_{pl} + W_{p2})/2$

52. a. Compute

$$I_{spav} = I_T/W_p$$

52. b. $I_{spav \text{ vac}} = I_{T \text{ vac}}/W_p$

53. Determine the maximum head end pressure $P_{h \text{ max}}$

Inert Weight Calculations

54. Calculate the maximum expected instantaneous pressure at the upper limit of initial grain temperature

$$P_{meop} = P_{h \text{ max}} [1 + n_1 (\sigma_p/P)] \exp[(\pi_p)_K \Delta T_i]$$

55. Calculate the minimum expected case material yield strength

$$S_{yc} = S_{yc \text{ nom}} [1 - n_2 (\sigma_{S_{yc}} / S_{yc})]$$

56. Compute the nominal thickness of the cylindrical portion of the case

$$\tau_{cc} = \psi_c P_{meop} D_{cc} / 2S_{yc}$$

57. Compute the weight of the case material in the cylindrical length including forward and aft segment joints

$$W_{cc} = \pi \tau_{cc} D_{cc} \delta_c L_{cc} [1 + (n_{seg} - 1)(40\tau_{cc} / L_{cc})]$$

Note: A joint is considered 4 nominal case thicknesses thick and 10 nominal case thicknesses long.

58. Compute the hoop-stress thickness requirement for the head end and aft end closures.

$$\tau_{cd} = \psi_c P_{meop} D_{cc} / 4 S_{yc}$$

59. Compute the weight of the head end dome

$$W_{ch} = 2.5 \pi D_{cc}^2 \delta_c \tau_{cd} / 2$$

Note: The 2.5 multiplying factor is an allowance for the igniter boss and forward thrust skirt attachment joint ("Y" joint). A similar but larger (4.5) allowance is made in the aft closure weight calculation for the nozzle and aft thrust skirt attachment joint. In case only one skirt is used, the dual allowance should still roughly approximate the net weight requirement because of the greater load carrying requirement placed on the single thrust reacting element.

60. Compute the estimated weight of the nozzle closure

$$W_{cn} = 4.5 \pi D_{cc} h_{cn} \tau_{cd} \delta_c / 2$$

61. Compute the total estimated weight of the motor case

$$W_c = W_{cc} + W_{ch} + W_{cn}$$

62. Compute the nozzle expansion ratio

$$\epsilon = A_e/A^*$$

63. Compute the estimated weight of the nozzle

$$W_n = K_1 D^{*2} [K_2 t_f \psi_a + (\epsilon - \epsilon^{1/2}) P_{meop} D^* \psi_s / S_{yn}] / [1 + (\sin \alpha_n) / 2]$$

64. Compute the estimated weight of the case insulation

$$\begin{aligned} W_{ins} = & t_f \psi_{ins} \delta_{ins} \pi D_{cc} \{ K_{en} [0.80 D_{cc} / 2 \\ & + (s + n_s) \tau_w / 2 \\ & + 0.15 (L_{cc} - \tau_w S - \tau_w n_s) / \psi_{ins}] \\ & + K_{en} 0.80 h_{cn} \} \end{aligned}$$

Note: Head and nozzle end domes are assumed exposed for 80 percent of t_p . The cylindrical length, exclusive of the areas exposed by circumferential slot burning, is assumed exposed for 15 percent of t_p with no safety factor. No allowance is made for final thermal protection of exposed areas for which an allowance may be included in W_a .

65. Compute the estimated weight of the liner

$$W_l = \tau_l \delta_l \pi D_{cc} [D_{cc} / 2 + L_{cc} + h_{cn}]$$

66. Compute the estimated total motor weight

$$W_m = W_c + W_n + W_{ins} + W_l + W_a + W_p$$

67. Compute the motor mass fraction

$$\zeta_M = W_p / W_M$$

68. Compute the total impulse to motor weight ratio I_T / W_M .

Burning Surfaces and Port Areas

The geometric relations described below are treated in the computer program as a major subroutine. It should be noted that end effects are treated by approximations both in the specification of input variables and in several instances in the mathematical relationships used.

A0.a. Select an option for the type of input.

If only tabular input, set $I_{op} = 1$

If only equation input, set $I_{op} = 2$

If a combination input, set $I_{op} = 3$

A0.b. Select an option for the grain configuration.

If only a circular perforated grain, set $G_{op} = 1$

If only a star grain, set $G_{op} = 2$

If a combination grain, set $G_{op} = 3$

A0.c. Select an option for the type of star.

If a standard star, set $S_{op} = 1$

If a truncated star, set $S_{op} = 2$

A0.d. Select an option for the grain arrangement.

If a star grain is at head end of the chamber and a circular perforated grain at nozzle end, set $Q_{op} = 1$

If circular perforated grains are at both ends of the chamber, set $Q_{op} = 2$

If a circular perforated grain is at the head end of the chamber and a star grain is at the nozzle end, set $Q_{op} = 3$

If star grains are at both ends of the chamber, set $Q_{op} = 4$

A0.e. Specify the number (n_T) of thrust termination passageways in the grain.

A0.f. Specify tabular input values of burning surface areas A_{bpT} , A_{bsT} , and A_{bnT} versus y . Set each equal to zero if $I_{op} = 2$.

- A0.g. If $I_{op} = 1$, specify the head end and nozzle end controlling port areas A_{pnT} versus y .
- A1. If $I_{op} = 1$, set $\Delta A_{bT} = A_{bpc} = A_{bps} = A_{bsc} = A_{bss} = A_{bnc} = A_{bns} = 0$
Go to step A51
- A2. If $I_{op} = 2$ or 3 Go to step A3a
- A3.a. If $G_{op} = 1$ or 3 Go to step A4
- A3.b. If $G_{op} = 2$, set $A_{bpc} = A_{bsc} = A_{bnc} = 0$
Go to step A7
- A4. Compute the burning surface areas of tubular grain segments:
- A4.a. For slots
- $$A_{bsc} = \pi S / [D_o^2 - (D_i + 2y)^2] / 4 \text{ for } A_{bsc} > 0$$
- $$A_{bsc} = 0 \text{ for } A_{bsc} \leq 0$$
- A4.b. For perforations
- If $2y + D_i > D_o$, set $A_{bnc} = A_{bpc} = 0$ Go to step A5
- If not and if $\theta_G \leq 5^\circ$ (See Figure II-1) compute
- $$L_{Gc} = L_{Gci} - y (\cot \theta_{cn} + \cot \theta_{ch}) \text{ and}$$
- $$A_{bpc} = \pi (D_i + 2y) [L_{Gc} - T_\ell - Sy] \text{ for } A_{bpc} > 0$$
- $$A_{bpc} = 0 \text{ for } A_{bpc} \leq 0, \text{ or}$$
- If $\theta_G > 5^\circ$, compute
- $$A_{bpc} = \pi (D_i + 2y) \{ L_{Gci} - y \cot \theta_{ch} - T_\ell - [S + \tan (\theta_G/2)] y \} \text{ for } A_{bpc} > 0$$
- $$A_{bpc} = 0 \text{ for } A_{bpc} \leq 0$$
- A4.c. For the nozzle end surface
- If $\theta_G \leq 5^\circ$, set $A_{bnc} = 0$

If $\theta_G > 5^\circ$, compute

$$A_{bnc} = \pi [L_{Gni} - y \cot (\theta_G + \theta_{cn}) - y \tan (\theta_G/2)] [D_i + \Delta D_i + y + L_{Gni} \sin \theta_G + y \csc (\theta_G + \theta_{cn}) \sin \theta_{cn}] \text{ for } A_{bnc} > 0$$

Note: the above equation represents the entire surface as conical although a non-conical surface area evolves if $\theta_G + \theta_{cn} > \pi/2$.

$$A_{bnc} = 0 \text{ for } A_{bnc} \leq 0$$

A5. Compute the controlling port areas

A5.a. If $Q_{op} = 1$ or 4 Go to step A5b.

If $Q_{op} = 2$ or 3 , compute

$$A_{ph} = \pi [D_i + 2 \sum_j \bar{r}_{hj} \Delta t_j]^2 / 4 \text{ for } A_{ph} \leq \pi D_o^2 / 4$$

$$A_{ph} = \pi D_o^2 / 4 \text{ for } A_{ph} > \pi D_o^2 / 4$$

A5.b. If $Q_{op} = 3$ or 4 Go to step A6

If $Q_{op} = 1$ or 2 , compute

$$A_{pn} = \pi [D_i + \Delta D_i + 2 \sum_j \bar{r}_{nj} \Delta t_j]^2 / 4 \text{ for } A_{pn} \leq \pi D_o^2 / 4$$

$$A_{pn} = \pi D_o^2 / 4 \text{ for } A_{pn} > \pi D_o^2 / 4$$

A6. If $G_{op} = 1$, Go to step A50

A7.a. If $G_{op} = 2$ or 3 and $S_{op} = 1$ Go to step A20

A7.b. If $G_{op} = 2$ or 3 and $S_{op} = 2$ Go to step A8

A8. Compute the star point half angle (See Figure II-3)

$$\theta_s = \pi / n_p$$

A9. Compute the approximate length of star sides

$$l_s = R_c - r_s - f - R_p$$

A10. Compute the angular location of the burning slot sides

$$\theta_{sl} = \theta_s - \arcsin [(f+y)/(R_p+y)]$$

A11. Compute the perimeter of the grain. (If $\theta_{sl} < 0$, Go to step A22)

A11.a. If $y \leq r_s$, compute

$$S_G = 2n_p \{ (R_p + y) \theta_{sl} + \ell_s - [(R_p + y) \cos (\theta_s - \theta_{sl}) - R_p] + \pi(f+y)/2 \}$$

Go to step A14

A11.b. If not, compute

$$\theta_c = \arcsin \{ [R_c^2 - (\ell_s + R_p)^2 - (f+y)^2] / 2(f+y)(\ell_s + R_p) \}$$

A12. If $\theta_c \geq 0$, compute

$$S_G = 2n_p \{ (R_p + y) \theta_{sl} + \ell_s - [(R_p + y) \cos (\theta_s - \theta_{sl}) - R_p] + (y+f) \theta_c \}$$

Go to step A14

A13.a. If $\theta_c < 0$ and $y < R_c - R_p$, compute

$$S_G = 2n_p \{ (R_p + y) \theta_{sl} + [R_c^2 - (f+y)^2]^{1/2} - [(R_p + y)^2 - (f+y)^2]^{1/2} \}$$

Go to step A14

A13.b. If $\theta_c < 0$ and $y \geq R_c - R_p$, set $S_G = 0$

A14. Compute the initial burning area of flat ends of grain.

$$A_{Gs} = \pi(R_c^2 - R_p^2) - n_p (\pi f^2/2 + 2\ell_s f)$$

A15. Compute burning slot end surface areas

$$A_{bss} = n_s [A_{Gs} - \sum_j \bar{S}_{Gj} \bar{r}_{avj} \Delta t_j]$$

Set $A_{bss} = 0$ if $A_{bss} < 0$ or $S_G = 0$

A16. Compute burning flat nozzle end surface area

$$A_{bns} = n_n [A_{Gs} - \sum_j \bar{S}_{Gj} \bar{r}_{nj} \Delta t_j]$$

Set $A_{bns} = 0$ if $A_{bns} < 0$ or $S_G = 0$

Go to step A31

A17,18,19 Not used.

A20. Compute the half angle of the standard star grain (See Fig. II-2).

$$\theta_s = \pi/n_p$$

A21. Compute the radial position of the fillet center

$$\ell_f = R_c - \tau_w - f$$

A22. Compute the grain perimeter (Equation A22a and A22b are taken from Reference 6, p. 303)

A22.a. If $(y+f)/\ell_f \leq (\sin \theta_f)/\cos(\theta_p/2)$ and $y < \tau_w$,

$$S_G = 2n_p \ell_f \{ (\sin \theta_f)/\sin(\theta_p/2) + [(y+f)/\ell_f][\pi/2 + \theta_s - \theta_p/2 \cot(\theta_p/2)] + \theta_s - \theta_f \} \quad \text{Go to step A23}$$

A22.b. If $(y+f)/\ell_f \leq (\sin \theta_f)/\cos(\theta_p/2)$ and $y \geq \tau_w$,

$$S_G = 2n_p \{ (y+f) \arcsin \left[\frac{[R_c^2 - \ell_f^2 - (y+f)^2]}{2\ell_f(y+f)} \right] - (\theta_s - \theta_p/2) \} \\ + (\ell_f \sin \theta_f)/\sin(\theta_p/2) - (y+f) \cot(\theta_p/2) \}$$

A22.c. If $(y+f)/\ell_f > (\sin \theta_f)/\cos(\theta_p/2)$ and $y < \tau_w$,

$$S_G = 2n_p \ell_f \{ [(y+f)/\ell_f][\theta_s + \arcsin(\frac{\ell_f}{y+f} \sin \theta_f)] + \theta_s - \theta_f \} \quad \text{Go to step A23}$$

A22.d. If $(y+f)/\ell_f > (\sin \theta_f)/\cos(\theta_p/2)$ and $y \geq \tau_w$,

$$S_G = 2n_p (y+f) \left\{ \theta_f + \arcsin \left(\frac{\ell_f}{y+f} \sin \theta_f \right) - \arcsin \cos \frac{[R_c^2 - \ell_f^2 - (y+f)^2]}{2\ell_f(y+f)} \right\} \text{ for } S_G > 0$$

$$S_G = 0 \text{ for } S_G \leq 0$$

A23. Compute the initial burning areas of slots and flat nozzle end.
(If $\theta_{s1} < 0$, Go to step A14)

$$A_{Gs} = \pi R_c^2 - n_p \ell_f^2 \{ \sin \theta_f [\cos \theta_f - (\sin \theta_f) \cot(\theta_p/2)] \\ + \theta_s - \theta_f + (2f/\ell_f) [(\sin \theta_f)/\sin(\theta_p/2) \\ + \theta_s - \theta_f + (f/2\ell_f)(\pi/2 + \theta_s - \theta_p/2 - \cot \frac{\theta_p}{2})] \}$$

A24. Compute the burning slot surfaces

$$A_{bss} = n_s [A_{Gs} - \sum_j \bar{S}_{Gj} \bar{r}_{avj} \Delta t_j] \text{ for } A_{bss} > 0$$

$$A_{bss} = 0 \text{ for } A_{bss} \leq 0$$

A25. Compute the burning flat nozzle end surface

$$A_{bns} = n_n [A_{Gs} - \sum_j \bar{S}_{Gj} \bar{r}_{nj} \Delta t_j] \text{ for } A_{bns} > 0$$

$$A_{bns} = 0 \text{ for } A_{bns} \leq 0$$

A26-30 Not used

A31. Compute the port burning surface areas

$$A_{bps} = [L_{Gsi} - T_\ell - y(n_s + n_n)] S_G \text{ for } A_{bps} > 0$$

$$A_{bps} = 0 \text{ for } A_{bps} \leq 0$$

A32-35 Not used

A36. Compute the controlling port areas of the grain segments

A36.a. If $Q_{op} = 2$ or 3

Go to step A36b

If $Q_{op} = 1$ or 4 , compute

$$A_{ph} = \pi R_c^2 - A_{Gs} + \sum_j \bar{S}_{Gj} \bar{r}_{hj} \Delta t_j \text{ for } A_{ph} < \pi R_c^2$$

$$A_{ph} = \pi R_c^2 \text{ for } A_{ph} \geq \pi R_c^2 \text{ or } S_G \leq 0$$

A36.b. If $Q_{op} = 1$ or 2

Go to step A50

If $Q_{op} = 3$ or 4 , compute

$$A_{pn} = \pi R_c^2 - A_{Gs} + \sum_j \bar{S}_{Gj} \bar{r}_{nj} \Delta t_j \text{ for } A_{pn} < \pi R_c^2$$

$$A_{pn} = \pi R_c^2 \text{ for } A_{pn} \geq \pi R_c^2 \text{ or } S_G \leq 0$$

A37-49 Not used

A50.a. If there are no thrust termination passageways in the grain ($N_T=0$),

Go to step A51

A50.b. If $n_T \neq 0$, compute change in A_b

$$\Delta A_{bT} = n_T \pi [(D_{Tp} + 2y)(l_{Tp} - y/\sin \theta_{Tp}) - (D_{Tp} + 2y)^2/4 + (y + D_{Tp}/2)(D_{Tp}/2)(1 - 1/\sin \theta_{Tp})] \text{ for } y \leq \tau_{Teff}$$

$$\Delta A_{bT} = 0 \text{ for } y \geq \tau_{Teff}$$

A51. Compute special equation surface area inputs B_{bp} , B_{bs} , and B_{bn}

Note: the B's are all set equal to zero in the present program but the zeros may be replaced by any functions of y desired simply by changing three program statements giving the B's

A52. Compute the total surface areas

$$A_{bp} = A_{bpT} + A_{bpc} + A_{bps} + \Delta A_{bT} + B_{bp}$$

$$A_{bs} = A_{bsT} + A_{bsc} + A_{bss} + B_{bs}$$

$$A_{bn} = A_{bnT} + A_{bnc} + A_{bns} + B_{bn}$$

Note: the present analysis treats A_{bp} and A_{bs} identically. They are kept separate, however, to permit possible alternate treatment.

A53. Compute the burning surface areas applicable to tailoff surface area calculations

If $z_w \geq 0$, set

$$A_{bm} = (A_{bp} + A_{bs} + A_{bn}) \Big|_y - \sum_{j=w}^j \Delta y_j / 2$$

$$A_{bto} = (A_{bp} + A_{bs} + A_{bn}) \Big|_y + (z_w/2) - \sum_{j=w}^j \Delta y_j / 2$$

If $z_w < 0$, set

$$A_{bm} = (A_{bp} + A_{bs} + A_{bn}) \Big|_y - (z_w/2) - \sum_{j=w}^j \Delta y_j / 2$$

$$A_{bto} = (A_{bp} + A_{bs} + A_{bn}) \Big|_y - \sum_{j=w}^j \Delta y_j / 2$$

IV. THE COMPUTER PROGRAM

This section contains the instructions for preparation and arrangement of the data cards. Also a complete listing of the program statement is given followed by flowcharts showing the sequence of computer operations.

Data Card Usage

The data formats have been established to allow the operator to look at the card and know which variables are represented on it. The format should be followed to insure correct reading of the inputs. The various data cards are as follows: (See Table IV-1 for the complete format)

- I. Number of configurations
- II. Initial zero values
- III. Propellant data
- IV a&b. Motor geometry (2 cards)
- V a&b. Performance data (2 cards)
- VI. Input, grain, etc.
- VII a&b. Data for tabular input (2 cards)
- VIII a&b. Tubular grain input (2 cards)
- IX. General star data
- X. Truncated star data
- XI. Standard star data
- XII. Termination port data
- XIII a-e. Data for weight calculations (5 cards)

Cards II-VI must accompany each and every configuration even if only one parameter changes. If INPUT = 1 or 3, a number of sets of data cards VII a&b are used. Consider a test run on three different configurations:

- a. A combination grain with only tabular inputs.
- b. A combination circular perforated and standard star grain with only equation inputs (no termination ports).
- c. A truncated star grain with two termination ports using both tabular and equation inputs.

The correct order for the data cards is:

<u>Configuration a</u>	<u>Configuration b</u>	<u>Configuration c</u>
1. I	12. II	21. II
2. II	13. III	22. III
3. III	14. IV a&b	23. IV a&b
4. IV a&b	15. V a&b	24. V a&b
5. V a&b	16. VI	25. VI
6. VII	17. VIII a&b	26. VII a&b (y=0)
7. VII a&b (y=0)	18. IX	27. IX
8. VII a&b (y=y ₁)	19. XI	28. X
9. VII a&b (y=y ₂)	20. XIII a-e	29. XII
10. VII a&b (y=y ₃)		30. VII a&b (y=y ₁)
11. XIII a-e		31. VII a&b (y=y ₂)
		32. VII a&b (y=y ₃)
		33. XIII a-e

Note that when tabular inputs are used alone, as in Configuration a., the sets of cards VII a&b (4 sets for Configuration a) giving the tabular inputs follow each other sequentially. When combination tabular and equation inputs are used, as in Configuration c., the sets of cards VII a&b for y=0 follows card VI and the remaining sets of cards VII a&b follow card XII or the next lower applicable number if XII does not apply. Note also that if a particular data card does not apply, no card is used. For example, X is omitted for Configuration b. and XI for Configuration c.

Program Listing

A complete program listing is presented in Table IV-2

Flowcharts

Flowcharts of the computer program are presented in Figures IV-1 through IV-3.

TABLE IV-1

DATA CARD FORMATS

cc1	cc10	cc20	cc30	cc40	cc50	cc60	cc70	cc80
				I				
NUMBER OF CONFIGURATIONS TO BE TESTED ---				II				
0.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.0				III				
RHO=--	A=--	N=--	ALPHA=--	BETA=--	MU=+-	E+--	CSTAR=--	
				IVa				
L=--	TAU=--	VCI=+-	E+--	VCF=+-	E+--	DE=--	DTI=--	
				IVb				
THETA=--		ALFAN=--	LTAP=--	XI=--	ZO=--			
				Va				
DELTAY=--	ZOUT=--	DPOUT=--	ZETAF=--	TB=--	HB=--	GAM=--		
				Vb				
RADER=--				VI				
INPUT=--	GRAIN=--	STAR=--	NT=--	ORDER=--				
				VIIa				
Y=--	ABPORT=+-	E+--	ABSLOT=+-	E+--	ABNOZ=+-	E+--		
				VIIb				
APHEAD=+-		E+--	APNOZ=+-	E+--				

TABLE IV-1 (Cont'd)

cc1	cc10	cc20	cc30	cc40	cc50	cc60	cc70	cc80
DO=---	DI=---	DELDI=---	VIIIa	S=---	THETAG=---			
LGCI=---	LGNI=---	THETCN=---	VIIIb	THETCH=---				
NS=---	LGSI=---	NP=---	IX	RC=---	FILLET=---	NN=---		
RP=---	TAUS=---		X					
THETAF=---	THETAP=---	TAUWS=---	XI					
LTP=---	DTP=---	THETTP=---	XII	TAUEFF=---				
PIPK=---	DTEMP=---	SIGMAP=---	XIIIa	SIGMAS=---	N1=---			
N2=---	SYCNOM=---	DCC=---	XIIIb	PSIC=---	DELC=---			
LCC=---	NSEG=---	HCN=---	XIIIc	SYNNOM=---	PSIS=---			
PSIA=---	K1=---	K2=---	XIIId	PSIINS=---	DELINS=---			
KEH=---	KEN=---	DLINER=---	XIIIe	TAUL=---	WA=---			

TABLE IV-2

```

C *****
C *
C *      SRM DESIGN AND PERFORMANCE ANALYSIS
C *      PREPARED AT AUBURN UNIVERSITY
C *      UNDER MCD. NO.6 TO COOPERATIVE AGREEMENT WITH
C *      NASA MARSHALL SPACE FLIGHT CENTER
C *
C *      ANALYSIS BY R.H. SFORZINI
C *      PROGRAMMING BY J.M. LYON
C *      AEROSPACE ENGINEERING DEPARTMENT
C *      AUGUST 1972
C *****
C REAL MGEN,MCIS,MNOZ,MN1,JRCCK,N,L,ME1,ME,ISP,ITOT,MU,MASS,ISPVAC
C REAL N1,N2,NSEG,K1,K2,KEH,KEN,NS,LCC,LTAP
C REAL M2,MDBAR,ISP2,ITVAC
C COMMON/CCNST1/ZW,AE,AT,THETA,ALFAN,G
C COMMON/CCNST2/CAPGAM,ME,BOT,ZETAF,TB,HB,GAM
C COMMON/CCNST3/S,NS
C COMMON/VARIA1/Y,T,DELY,DELTAT,PCNCZ,PHEAD,RNOZ,RHEAD,SUMAB,PHMAX
C COMMON/VARIA2/ABPORT,ABSLOT,ABNCZ,APHEAD,APNOZ,DADY,ABP2,ABN2,ABS2
C COMMON/VARIA3/ICT,ITVAC,JROCK,ISP,ISPVAC,MCIS,MNCZ,SG,SUMT
C COMMON/VARIA4/RNT,RHT,SUM2,R1,R2,R3,RHAVE,RNAVE,RBAR,YB,KOUNT,TL
C COMMON/VARIA5/ABMAIN,ABTO,SUMCY
C READ(5,500) NRLNS
C *****
C *      READ IN THE NUMBER OF CONFIGURATIONS TO BE TESTED
C *
C *****
C 500 FORMAT(42X,I2)
C      DO 901 I=1,NRLNS
C      WRITE(6,602) I
C 602 FORMAT(1F1,42X,'CONFIGURATION NUMBER ',I2)
C      READ(5,499) SUMCY,ANS,ZW,Y,T,DELTAT,RNOZ,RHEAD,SUMAB,PHMAX,SUM2,IT
C      ICT,RHT,RNT,R1,R2,R3,RHAVE,RNAVE,RBAR,ITVAC,SUMT
C *****
C *      SET INITIAL VALUES OF SELECTED VARIABLES EQUAL TO ZERO
C *      ***NOTE*** THESE VALUES MUST BE ZEROED AT THE BEGINNING OF
C *      EACH CONFIGURATION RUN
C *****
C 499 FORMAT(22F3.1)
C      READ(5,501) RHC,A,N,ALPHA,BETA,MU,CSTAR
C *****
C *      READ IN BASIC PROPELLANT CHARACTERISTICS
C *
C *      RHC IS THE DENSITY OF THE PROPELLANT IN SLUGS/IN**3
C *      A IS THE BURNING RATE COEFFICIENT
C *      N IS THE BURNING RATE EXPONENT
C *      ALPHA AND BETA ARE THE CONSTANTS IN THE ERCSIVE BURNING
C *      RELATION OF ROBILLARD AND LENIOR
C *      MU IS THE VISCOSITY OF THE PROPELLANT GASES

```


TABLE IV-2 (Cont'd)

```

C *      CSTAR IS THE CHARACTERISTIC EXHAUST VELOCITY IN FT/SEC      *
C *****
501 FORMAT(4X,F8.6,3X,F6.4,3X,F5.3,7X,F4.1,6X,F5.1,4X,E11.4,7X,F5.0)
    WRITE(6,603) RHO,A,N,ALPHA,BETA,MU,CSTAR
603 FORMAT(//,20X,'PROPELLANT CHARACTERISTICS',/,13X,'RHO= ',F8.6,/,1
    13X,'A= ',F6.4,/,13X,'N= ',F5.3,/,13X,'ALPHA= ',F4.1,/,13X,'BETA= '
    2,F5.1,/,13X,'MU= ',1PE11.4,/,13X,'CSTAR= ',1PE11.4)
    READ(5,502) L,TAU,VCI,VCF,DE,DTI,THETA,ALFAN,LTAP,XT,ZC
C *****
C *      READ IN BASIC MOTOR DIMENSIONS                               *
C *
C *      L IS THE TOTAL LENGTH OF THE GRAIN IN INCHES                 *
C *      TAU IS THE AVERAGE WEB THICKNESS OF THE CONTROLLING GRAIN   *
C *      LENGTH IN INCHES                                             *
C *      VCI IS THE INITIAL CHAMBER VOLUME IN IN**3                  *
C *      VCF IS THE FINAL CHAMBER VOLUME IN IN**3                    *
C *      DE IS THE DIAMETER OF THE NOZZLE EXIT IN INCHES             *
C *      DTI IS THE INITIAL DIAMETER OF THE NOZZLE THROAT IN INCHES  *
C *      THETA IS THE CANT ANGLE OF THE NOZZLE WITH RESPECT TO THE   *
C *      MOTOR AXIS IN RADIANS                                         *
C *      ALFAN IS THE EXIT HALF ANGLE OF THE NOZZLE IN RADIANS        *
C *      LTAP IS THE LENGTH OF THE GRAIN AT THE NOZZLE END HAVING    *
C *      ADDITIONAL TAPER NOT REPRESENTED BY ZC IN INCHES            *
C *      XT IS THE DIFFERENCE IN WEB THICKNESS ASSOCIATED WITH LTAP  *
C *      ZC IS THE INITIAL DIFFERENCE BETWEEN WEB THICKNESSES AT THE *
C *      HEAD AND AFT ENDS OF THE CONTROLLING GRAIN LENGTH           *
C *****
502 FORMAT(2X,F5.0,5X,F5.2,5X,E11.4,5X,E11.4,4X,F6.2,5X,F6.2,/,14X,F6.
    14,7X,F6.4,7X,F6.2,5X,F5.2,5X,F5.2)
    WRITE(6,604) L,TAU,VCI,VCF,DE,DTI,THETA,ALFAN,LTAP,XT,ZC
604 FORMAT(//,20X,'BASIC MOTOR DIMENSIONS',/,13X,'L= ',F5.0,/,13X,'TAU
    1= ',F5.2,/,13X,'VCI= ',1PE11.4,/,13X,'VCF= ',1PE11.4,/,13X,'DE= ',
    21PE11.4,/,13X,'DTI= ',1PE11.4,/,13X,'THETA= ',1PE11.4,/,13X,'ALFAN=
    3 ',1PE11.4,/,13X,'LTAP= ',1PE11.4,/,13X,'XT= ',1PE11.4,/,13X,'ZC=
    4 ',1PE11.4)
    READ(5,503) DELTAY,XOUT,DPCUT,ZETAF,TB,HB,GAM,RADER
C *****
C *      READ IN BASIC PERFORMANCE CONSTANTS                           *
C *
C *      DELTAY IS THE DESIRED BURN INCREMENT DURING TAILOFF IN INCHES *
C *      XOUT IS THE DISTANCE BURNED IN INCHES AT WHICH THE PROPELLANT *
C *      BREAKS UP                                                    *
C *      DPCUT IS THE DEPRESSURIZATION RATE IN LB/IN**3 AT WHICH THE  *
C *      PROPELLANT IS EXTINGUISHED                                  *
C *      ZETAF IS THE THRUST LOSS COEFFICIENT                          *
C *      TB IS THE ESTIMATED BURN TIME IN SECONDS                     *
C *      HB IS THE ESTIMATED BURNOUT ALTITUDE IN FEET                 *
C *      GAM IS THE RATIO OF SPECIFIC HEATS FOR THE PROPELLANT GASES  *

```

TABLE IV-2 (Cont'd)

```

C *      RADER IS THE RADIAL EROSION RATE OF THE NOZZLE THROAT IN      *
C *      INCHES/SEC                                                    *
C *****
503 FORMAT(7X,F5.3,6X,F7.2,7X,F7.2,7X,F4.3,4X,F5.1,4X,F7.0,5X,F5.3,/,8
1X,F6.4)
WRITE(6,606) DELTAY,XCUT,DPCUT,ZETAF,TB,HB,GAM,RADER
606 FORMAT(/,15X,'BASIC PERFORMANCE CONSTANTS',/,13X,'DELTAY= ',F5.3,
1/,13X,'XCUT= ',F7.2,/,13X,'DPCUT= ',F7.2,/,13X,'ZETAF= ',F5.3,/,13
2X,'TB= ',F5.1,/,13X,'HB= ',F7.0,/,13X,'GAM= ',F5.3,/,13X,'RADER= '
3,F6.4)
MN1=.85
ME1=7.0
G=32.2
Z=20
S=0.0
NS=0.0
KOUNT=0
ABMAIN=0.0
ABTC=0.0
DELY=DELTAY
TCP=GAM+1.
BOT=GAM-1.
VC=VCI
ZAP=TCP/(2.*BCT)
CAPGAM=SQRT(GAM)*(2./TCP)**ZAP
AE=3.14159*CE**2/4.
1 IF(XT.LE.0.0) TL=0.0
IF(XT.LE.0.0) GC TO 40
TL=(Y-TAU+XT+Z/2.)*LTAP/XT
IF(TL.LE.0.0) TL=0.0
IF(TL.GE.LTAP) TL=LTAP
40 DT=DTI+2.*(RADER*T)
AT=3.14159*CT**2/4.
CALL AREAS
IF(ABS(ZW).GT.0.0) GO TO 20
IF(SUMAB.LE.0.0) GO TO 31
X=(ABPCRT+ABSLCT)/SUMAB
90 MNOZ=AT*X/APNCZ*(2.*(1.+BOT/2.*MN1*MN1)/TCP)**ZAP
IF(ABS(MNOZ-MN1).LE.0.002) GO TO 2
MN1=MNOZ
GC TO 90
2 VNCZ=GAM*CTAR*MNOZ*SQRT(((2./TCP)**(TOP/BCT))/(1.+BOT/2.*MNOZ*MNO
1Z))
PRAT=(1.+BOT/2.*MNOZ*MNOZ)**(-GAM/BCT)
JRCK=AT/APNCZ
SUMYA=DELY*(ABP2+ABN2+ABS2)
IF(Y.EQ.0.0) SUMYA=0.0
VC=VC+SUMYA

```

TABLE IV-2 (Cont'd)

```

IF(Y.GT.0.0) GC TO 11
PCNCZ=(A*RHC*CSTAR*SUMPB/AT)**(1./(1.-N))*(1.+(CAPGAM*JROCK)**2/2.
1)**(N/(1.-N))
PCN=PCNOZ
MCIS=AT*PCNCZ/CSTAR
P2=PCNCZ
PCNCZ2=PCNOZ
PNCZ=PRAT*PCNCZ
P4=2.*MDIS*VNCZ/(APHEAD+APNOZ)+PNCZ
5 PNCZ=PRAT*PCNCZ
PHEAD=2.*MDIS*VNCZ/(APHEAD+APNOZ)+PNOZ
RHEAD=A*PHEAD**N
ZIT=MCIS*X/APNCZ
RN1=RHEAD
PHEAD2=PHEAD
3 RNOZ=RN1-(((RN1-A*PNCZ**N-ALPHA*ZIT**.8/(L**.2*EXP(BETA*RN1*RHO/ZIT
1))))/(1.+ALPHA*ZIT**.8*BETA*RHC/ZIT/(L**.2*EXP(BETA*RN1*RHO/ZIT))))
IF(ABS(RN1-RNCZ).LE.0.002) GO TO 4
RN1=RNOZ
GO TO 3
4 AVE1=(RHEAD+RNCZ)/2.
IF(Y.GT.0.0) GC TO 7
RN2=RNCZ
RH2=RHEAD
PCNJ=PCNCZ
DPCDY=0.0
AVE2=AVE1
7 RNAVE=(RNOZ+RN2)/2.
RHAVE=(RHEAD+RH2)/2.
MGEN=RHO/2.*((RNOZ+RHEAD)*(ABPORT+ABSLOT))+2.*A*PCNOZ**N*ABNCZ
DRDY=(AVE1-AVE2)/DELY
RBAR=(AVE1+AVE2)/2.
GMAX=1.002*MCIS
GMIN=0.998*MDIS
IF(Y.GT.0.0) GC TO 12
GMAX=1.001*MCIS
GMIN=0.999*MDIS
IF(MGEN.GE.GMIN.AND.MGEN.LE.GMAX) GC TO 6
MCIS=MGEN
PCNOZ=MDIS*CSTAR/AT
GC TO 5
6 RE=2.*MDIS*X*L/((APNCZ+APHEAD)*MU)
IF(Y.LE.0.0) WRITE(6,101) RE
101 FORMAT(13X,'INITIAL REYNOLDS NUMBER= ',1PE11.4)
PCNJ=PCNCZ
17 ME=SQR(2./BCT*(TOP/2.*(AE*ME1/AT)**(1./ZAP)-1.))
IF(ABS(ME-ME1).LE.0.002) GC TO 9
ME1=ME

```

TABLE IV-2 (Cont'd)

```

GO TO 17
9 CALL OUTPUT
10 IF(Y.LE..05*TAU) GO TO 16
   SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12.
   MASS=.01*MDIS
   ANS4=Y+10.0*DELTAY
   IF(KOUNT.GT.0) GO TO 16
   IF(ABS(SINK1).LE.MASS.AND.ANS4.LE.ANS-XT) GO TO 18
   GO TO 16
18 DELY=10.*DELTAY
   GO TO 55
16 DELY=DELTAY
55 DELTAT=2.*DELY/(RHAVE+RNAVE)
   Z=Z+DELTAT*(RNAVE-RHAVE)
   Y=Y+DELY
   T=T+DELTAT
   SUM2=SUMAB
   RN2=RN0Z
   RH2=RHEAD
   AVE2=AVE1
   GO TO 1
11 MDIS=AT*PCNOZ/CSTAR
   GO TO 5
12 DPCDY=(PHEAD2+PCNOZ2)/((RNAVE+RHAVE)*DRDY+(PHEAD2+PCNOZ2)/((ABP2+AB
   IN2+ABS2)*2.)*DACY
   IF(ABS(DPCDY).GE.DPOUT.CR.Y.GE.XCUT) GO TO 25
   SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12.+(PHEAD2+PCNOZ2)/2.*(RNAV
   IE+RHAVE)/2.*(ABP2+ABN2+ABS2)/(12.*(CSTAR*CAPGAM)**2)
   STUFF=PGEN-SINK1
   IF(STUFF.GE.GMIN.AND.STUFF.LE.GMAX) GO TO 14
   MDIS=STUFF
   PCNOZ=MDIS*CSTAR/AT
   GO TO 5
14 P1=PCNOZ
   PCNJ=PCNOZ
   PCNCZ2=(P1+P2)/2.
   P2=PCNCZ
   P3=PHEAD
   PHEAD2=(P3+P4)/2.
   P4=PHEAD
   ANS=TAU-ABS(Z/2.)
   IF(Y.LT.ANS) CALL OUTPUT
   IF(Y.LT.ANS) GO TO 10
19 ZW=Z
   YW=Y
   SUMBA=SUMAB
   P1=PCNOZ
   RH2=RHEAD

```

TABLE IV-2 (Cont'd)

```

RN2=RNCZ
RAVE=AVE1
ABMAIN=SUMAB
ABTC=0.0
WRITE(6,51)
51 FORMAT(48X,'*****',/,48X,'****TAIL OFF BEGINS***
1*',/,48X,'*****',//)
20 ANS2=IAU+ABS(ZW/2.)
KOUNT=KOUNT+1
DELYW=DELTAY
DY2=DELYW
IF(ZW) 32,32,33
32 IF(Y.LT.ANS2.AND.ABS(ZW).GT.DY2) GO TO 211
SUMAB=ABMAIN
GO TO 31
211 SUMDY=SUMDY+DELYW
SUMAB=(1.+SUMDY/ZW-DELYW/(2.*ZW))*ABTC-(SUMDY/ZW-DELYW/(2.*ZW))*AB
MAIN
GO TO 31
33 IF(Y.LT.ANS2.AND.ZW.GT.DY2) GO TO 21
SUMAB=ABTC
GO TO 31
21 SUMDY=SUMDY+DELYW
SUMAB=(1.-SUMDY/ZW+DELYW/(2.*ZW))*ABMAIN+(SUMDY/ZW-DELYW/(2.*ZW))*
ABTC
31 IF(SUMAB.LE.0.C) PONCZ=PONCZ/2.
IF(SUMAB.LE.0.C) GO TO 25
PONCZ=(A*RHC*CSTAR*SUMAB/AT)**(1./(1.-N))
MDIS=A*PONCZ/CSTAR
ABAVE=(SUMAB+SUMBA)/2.
SUMYA=DELY*ABAVE
VC=VC+SUMYA
DADY=(SUMAB-SUMBA)/DELY
PBAR=(P1+PONCZ)/2.
SUMBA=SUMAB
22 DPCCDY=PBAR/(1.-N)*1./ABAVE*DADY
IF(PONCZ.LE.30.C) GO TO 25
RNCZ=A*PONCZ**N
RHEAD=RNCZ
RBAR=(RHEAD+RAVE)/2.
NGEN=RHO*(RNCZ+RHEAD)/2.*SUMAB
GMAX=1.002*MDIS
GMIN=0.998*MDIS
SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCCDY/12.
STUFF=NGEN-SINK1
IF(STUFF.GE.GMIN.AND.STUFF.LE.GMAX) GO TO 23
MDIS=STUFF
PONCZ=PONJ+DPCCDY*DELY

```

TABLE IV-2 (Cont'd)

```

      IF(PONCZ.LE.0.C) PONCZ=0.0
      PBAR=(P1+PCNCZ)/2.
      GC TO 22
23  RFAVE=(RF2+RHEAD)/2.
      RNAVE=(RN2+RNCZ)/2.
      RH2=RHEAD
      RN2=RNCZ
      PHEAD=PONCZ
      RAVE=RHEAD
      P1=PONCZ
      PCNJ=PCNCZ
      IF(ABS(DPCDY).GE.DPCUT) GO TO 25
      IF(Y.GE.XOUT) GC TO 25
      CALL OUTPUT
      GC TO 10
25  SUMAB=0.0
      RHEAD=C.0
      RACZ=RHEAD
      PHEAD=PONCZ
      WRITE(6,318)
318 FORMAT(44X,'*****BEGIN HALF SECOND TRACE*****',//)
      CALL OUTPUT
      TIME=T
      DELTAT=.5
      TIM=TIME+5.
      PHT=PHEAD
      PONT=PCNCZ
      SG=0.0
29  T=T+DELTAT
      PHEAD=PHT/EXP(CAPGAM**2*AT*CSTAR/VCF*(T-TIME)*12.)
      PCNCZ=PHEAD
      MDIS=PCNCZ*AT/CSTAR
      Y=Y+.5*RHEAD
      CALL OUTPUT
28  IF(T.LT.TIM.AND.PHEAD.GE.30.0) GC TO 29
100 WP1=G*SUMMT
      WP2=RHC*(VC-VC1)*G
      WP=(WP1+WP2)/2.
      ISP=ITCT/WP
      ISPVAC=ITVAC/WP
      WRITE(6,102) WP1,WP2,WP,PHMAX,ISP,ISPVAC,ITOT,ITVAC
102 FORMAT(13X,'WP1= ',1PE11.4,/,13X,'WP2= ',1PE11.4,/,13X,'WP= ',1PE1
11.4,/,13X,'PHMAX= ',1PE11.4,/,13X,'ISP= ',1PE11.4,/,13X,'ISPVAC= '
2,1PE11.4,/,13X,'ITCT= ',1PE11.4,/,13X,'ITVAC= ',1PE11.4)
      READ(5,600) PIFK,DTEMP,SIGMAP,SIGMAS,N1,N2,SYCNOM,DCC,PSIC,DELC,LC
1C,NSEG,HCN,SYNCEM,PSIS,PSIA,K1,K2,PSIINS,DELINS,KEH,KEN,DLINER,TAU
2L,WA
C  *****

```

TABLE IV-2 (Cont'd)

```

C * READ IN BASIC PROPERTIES REQUIRED FOR WEIGHT CALCULATIONS *
C *
C * PIPK IS THE TEMPERATURE SENSITIVITY COEFFICIENT OF PRESSURE *
C * AT CONSTANT K *
C * DTEMP IS THE MAX EXPECTED INCREASE IN TEMPERATURE ABOVE *
C * CONDITIONS UNDER WHICH MAIN TRACE WAS CALCULATED IN *
C * DEGREES FAHRENHEIT *
C * SIGMAP IS THE VARIATION IN PHMAX *
C * SIGMAS IS THE VARIATION IN CASE MATERIAL YIELD STRENGTH *
C * N1 IS THE NUMBER OF STANDARD DEVIATIONS IN PHMAX TO BE USED *
C * AS A BASIS FOR DESIGN *
C * N2 IS THE NUMBER OF STANDARD DEVIATIONS IN SY TO BE USED AS *
C * A BASIS FOR DESIGN *
C * SYNCNM IS THE NOMINAL YIELD STRENGTH OF THE CASE MATERIAL *
C * IN LBS/INCH *
C * DCC IS THE ESTIMATED MEAN DIAMETER OF THE CASE IN INCHES *
C * PSIC IS THE SAFETY FACTOR ON THE CASE THICKNESS *
600 FORMAT(7X,F5.4,9X,F6.2,10X,F5.3,10X,F5.3,6X,F5.2,/,5X,F5.2,10X,F10
1.2,7X,F6.2,8X,F5.2,8X,F5.3,/,6X,F7.2,8X,F3.0,7X,F4.1,10X,F10.2,8X,
2F5.2,/,7X,F5.2,6X,F6.4,6X,F6.4,10X,F5.2,10X,F6.4,/,6X,F6.4,7X,F6.4
3,10X,F6.4,8X,F6.4,6X,F7.2)
C * DELC IS THE SPECIFIC WEIGHT OF THE CASE MATERIAL IN LBS/IN**3 *
C * LCC IS THE LENGTH OF THE CYLINDRICAL PORTION OF THE CASE *
C * INCLUDING FORWARD AND AFT SEGMENTS IN INCHES *
C * NSEG IS THE NUMBER OF CASE SEGMENTS *
C * HCN IS THE AXIAL LENGTH OF THE NOZZLE CLOSURE IN INCHES *
C * SYNNOM IS THE NOMINAL YIELD STRENGTH OF THE NOZZLE MATERIAL *
C * IN LBS/INCH *
C * PSIS IS THE SAFETY FACTOR ON THE NOZZLE STRUCTURAL MATERIAL *
C * PSIA IS THE SAFETY FACTOR ON THE NOZZLE ABLATIVE MATERIAL *
C * K1 AND K2 ARE EMPIRICAL CONSTANTS IN THE NOZZLE WT. EQUATION *
C * PSIINS IS THE SAFETY FACTOR ON NOZZLE INSULATION *
C * DELINS IS THE SPECIFIC WEIGHT OF THE INSULATION IN LBS/IN**3 *
C * KEH IS THE EROSION RATE OF INSULATION TAKEN CONSTANT *
C * EVERYWHERE EXCEPT AT THE NOZZLE CLOSURE IN IN/SEC *
C * KEN IS THE EROSION RATE OF INSULATION AT THE NOZZLE CLOSURE *
C * IN IN/SEC *
C * DLINER IS THE SPECIFIC WEIGHT OF THE LINER IN LBS/IN**3 *
C * TAUL IS THE THICKNESS OF THE LINER IN INCHES *
C * WA IS ANY ADDITIONAL WEIGHT NOT CONSIDERED ELSEWHERE IN LBS *
C *
C *****
WRITE(6,610) PIPK,DTEMP,SIGMAP,SIGMAS,N1,N2,SYNNOM,DCC,PSIC,DELC,L
1CC,NSEG,HCN,SYNNOM,PSIS,PSIA,K1,K2,PSIINS,DELINS,KEH,KEN,DLINER,TA
2UL,WA
610 FORMAT(/,2CX,'INERT WEIGHT INPUTS',/,13X,'PIPK= ',1PE11.4,/,13X,'
1DTEMP= ',1PE11.4,/,13X,'SIGMAP= ',1PE11.4,/,13X,'SIGMAS= ',1PE11.4
2,/,13X,'N1= ',1PE11.4,/,13X,'N2= ',1PE11.4,/,13X,'SYNNOM= ',1PE11.
34,/,13X,'DCC= ',1PE11.4,/,13X,'PSIC= ',1PE11.4,/,13X,'DELC= ',1PE1

```

TABLE IV-2 (Cont'd)

```

41.4,/,13X,'LCC= ',1PE11.4,/,13X,'NSEG= ',1PE11.4,/,13X,'HCN= ',1PE
511.4,/,13X,'SYNCOM= ',1PE11.4,/,13X,'PSIS= ',1PE11.4,/,13X,'PSIA=
6',1PE11.4,/,13X,'K1= ',1PE11.4,/,13X,'K2= ',1PE11.4,/,13X,'PSIINS=
7 ',1PE11.4,/,13X,'DELINS= ',1PE11.4,/,13X,'KEF= ',1PE11.4,/,13X,'K
8EN= ',1PE11.4,/,13X,'CLINER= ',1PE11.4,/,13X,'TAUL= ',1PE11.4,/,13
9X,'WA= ',1PE11.4)
PMEOP=PHMAX*(1.+N1*SIGMAP)*EXP(PIPK*DTEMP)
SYC=SYCNOM*(1.-N2*SIGMAS)
TAUCC=PSIC*PMEOP*DCC/(2.*SYC)
WCC=3.14159*TAUCC*DCC*DELC*LCC*(1.+(NSEG-1.)*(40.*TAUCC/LCC))
TAUCD=TAUCC/2.
WCH=2.5*3.14159/2.*DCC**2*TAUCC*DELC
WCN=4.5*3.14159/2.*DCC*HCN*TAUCC*DELC
WC=WCC+WCH+WCN
EPSIL=AE/AT
DT=2.*SQRT(AT/3.14159)
WN=K1*DT**2/(1.+5*SIN(ALFAN))*((EPSIL-SQRT(EPSIL))*PMEOP*DT*PSIS/
1SYNOM+K2*T*PSIA)
WINS=T*PSIINS*DELINS*DCC*3.14159*(KEF*(DCC*.40+(S+NS)*TAU/2.+0.15/
1PSIINS*(LCC-TAL*(S+NS)))+KEN*.80*HCN)
WL=TAUL*DLINER*3.14159*DCC*(DCC/2.+LCC+HCN)
WP=WC+WN+WINS+WL+WA+WP
ZETAM=WP/WM
RATIO=ITCT/WM
WRITE(6,605)
605 FORMAT(///,42X,'MOTOR WEIGHT CALCULATIONS')
WRITE(6,601) PMEOP,TAUCC,WC,WN,WINS,WL,W,M,ZETAM,RATIO
601 FORMAT(13X,'MAX EXPECTED PRESSURE= ',1PE11.4,/,13X,'CYLINDRICAL CA
1SE THICKNESS= ',1PE11.4,/,13X,'CASE WT= ',1PE11.4,/,13X,'NOZZLE WT
2= ',1PE11.4,/,13X,'INSULATION WT= ',1PE11.4,/,13X,'LINER WT= ',1PE
311.4,/,13X,'TOTAL MOTOR WT= ',1PE11.4,/,13X,'ZETAM= ',1PE11.4,/,13
4X,'RATIO OF ITCT TO WM= ',1PE11.4)
901 CONTINUE
STOP
END

```


TABLE IV-2 (Cont'd)

```

SUBROUTINE AREAS
C *****
C * SUBROUTINE AREAS CALCULATES BURNING AREAS AND PORT AREAS FOR *
C * CIRCULAR PERFORATED (C.P.) GRAINS AND STAR GRAINS OR FOR A *
C * COMBINATION OF C.P. AND STAR GRAINS *
C *****
      INTEGER STAR, GRAIN, CRDER
      REAL MGEN, MDIS, MNOZ, MNI, JRCK, N, L, ME1, ME, ISP, ITOT, MU, MASS, ISPVAC
      REAL LGCI, LGNI, NS, AN, AP, LGS1, NT, LTP, LGC, LS, LF
      REAL M2, MDBAR, ISP2, ITVAC
      COMMON/CONST1/ZW, AE, AT, THETA, ALFAN, G
      COMMON/CONST3/S, NS
      COMMON/VARIA1/Y, T, DELY, DELTAT, PCNOZ, PHEAD, RNOZ, RHEAD, SUMAB, PHMAX
      COMMON/VARIA2/ABPORT, ABSLOT, ABNOZ, APHEAD, APNOZ, DADY, ABP2, ABN2, ABS2
      COMMON/VARIA3/ITOT, ITVAC, JRCK, ISP, ISPVAC, MDIS, MNOZ, SG, SUMT
      COMMON/VARIA4/RNT, RHT, SUM2, R1, R2, R3, RHAVE, RNAVE, RBAR, YB, KOUNT, TL
      COMMON/VARIA5/ABMAIN, ABTO, SUMDY
      ABPC=0.0
      ABNC=0.0
      ABSC=0.0
      ABPS=0.0
      ABNS=0.0
      ABSS=0.0
      DABT=0.0
      EBP=0.0
      EBN=0.0
      EBS=0.0
      SG=0.0
      ANUM=3.14159/4.
      RNT=RNT+RNOZ*DELTAT
      RHT=RHT+RHEAD*DELTAT
1  IF(ABS(ZW).LE.C.0) K=0
   IF(ABS(ZW).GT.C.0) K=1
   YB=Y
   IF(K.EQ.1) Y=YB-SUMDY/2.
2  IF(K.EQ.2) Y=YB+ABS(ZW)/2.-SUMDY/2.
   IF(Y.LE.C.0) READ(5,500) INPUT, GRAIN, STAR, NT, CRDER
C *****
C * READ THE TYPE OF INPUT FOR THE PROGRAM AND THE BASIC GRAIN *
C * CONFIGURATION AND ARRANGEMENT *
C * VALUES FOR INPUT ARE *
C * 1 FOR ONLY TABULAR INPUT *
C * 2 FOR ONLY EQUATION INPUTS (EQUATIONS ARE BUILT *
C * INTO THE SUBROUTINE) *
C * 3 FOR A COMBINATION OF 1 AND 2 *
C * VALUES FOR GRAIN ARE *
C * 1 FOR STRAIGHT C.P. GRAIN *
C * 2 FOR STRAIGHT STAR GRAIN *

```

TABLE IV-2 (Cont'd)

```

C *          3 FOR COMBINATION OF C.P. AND STAR GRAINS *
C *          VALUES FOR STAR ARE *
C *          0 FOR STRAIGHT C.P. GRAIN *
C *          1 FOR STANDARD STAR *
C *          2 FOR TRUNCATED STAR *
C *          VALUES FOR NT ARE *
C *          0 IF THERE ARE NO TERMINATION PORTS *
C *          X WHERE X IS THE NUMBER OF TERMINATION PORTS *
C *          VALUES OF ORDER ESTABLISH HOW A COMBINATION C.P. AND STAR *
C *          GRAIN IS ARRANGED *
C *          1 IF DESIGN IS STAR AT HEAD END AND C.P. AT NOZZLE *
C *          2 IF DESIGN IS C.P. AT HEAD END AND C.P. AT NOZZLE *
C *          3 IF DESIGN IS C.P. AT HEAD END AND STAR AT NOZZLE *
C *          4 IF DESIGN IS STAR AT HEAD END AND STAR AT NOZZLE *
C *          ***NOTE*** IF GRAIN=1, VALUE OF ORDER MUST BE 2 *
C *          ***NOTE*** IF GRAIN=2, VALUE OF ORDER MUST BE 4 *
C *****
500 FORMAT(9X,I2,9X,I2,8X,I2,6X,F4.0,9X,I2)
    IF(Y.LE.0.0) WRITE(6,607)
607 FORMAT(/,20X,'GRAIN CONFIGURATION')
    IF(Y.LE.0.0) WRITE(6,600) INPUT,GRAIN,STAR,NT,ORDER
600 FORMAT(13X,'INPUT= ',I2,/,13X,'GRAIN= ',I2,/,13X,'STAR= ',I2,/,13X
1,'NT= ',F4.0,/,13X,'ORDER= ',I2,/)
    IF(INPUT.EQ.2) GO TO 12
    IF(Y.LE.0.0) GO TO 6
    IF(YT.LE.Y.AND.K.LT.2) GO TO 8
9 DENOM=YT-YT2
  SLOPE1=(ABPK-ABPK2)/DENOM
  SLOPE2=(ABSK-ABSK2)/DENOM
  SLOPE3=(ABNK-ABNK2)/DENOM
  SLOPE4=(APHK-APHK2)/DENOM
  SLOPE5=(APNK-APNK2)/DENOM
  B1=ABPK-SLOPE1*YT
  B2=ABSK-SLOPE2*YT
  B3=ABNK-SLOPE3*YT
  B4=APHK-SLOPE4*YT
  B5=APNK-SLOPE5*YT
  ABPT=SLOPE1*Y+B1
  ABST=SLOPE2*Y+B2
  ABNT=SLOPE3*Y+B3
  APHT=SLOPE4*Y+B4
  APNT=SLOPE5*Y+B5
  IF(INPUT.EQ.3) GO TO 3
  GO TO 52
6 READ(5,507) YT,ABPK,ABSK,ABNK,APHK,APNK
C *****
C *          READ IN TABLLAR VALLES FOR Y=0.0 (NCT REQUIRED IF INPUT=2) *
C *

```

TABLE IV-2 (Cont'd)

```

C *      ABPK IS THE BURNING AREA IN THE PORT IN IN**2      *
C *      ABSK IS THE BURNING AREA IN THE SLCTS IN IN**2      *
C *      ABNK IS THE BURNING AREA IN THE NOZZLE END IN IN**2  *
C *      APHK IS THE PORT AREA AT THE HEAD END IN IN**2      *
C *      APNK IS THE PORT AREA AT THE NOZZLE END IN IN**2     *
C *****
507 FORMAT(6X,F6.2,10X,E11.4,10X,E11.4,8X,E11.4,/,22X,E11.4,9X,E11.4)
    WRITE(6,610)
610 FORMAT(/,13X,'TABULAR VALUES FOR YT EQUAL ZERO READ IN')
    ABPT=ABPK
    ABST=ABSK
    ABNT=ABNK
    APHT=APHK
    APNT=APNK
    YT2=YT
    IF(INPLT.EQ.3) GO TO 3
    GO TO 52
8  YT2=YT
    ABPK2=ABPK
    ABNK2=ABNK
    ABSK2=ABSK
    APHK2=APHK
    APNK2=APNK
    READ(5,505) YT,ABPK,ABSK,ABNK,APHK,APNK
C *****
C *      READ IN TABULAR VALUES FOR Y=Y      (NOT REQUIRED FOR INPUT=2) *
C *****
505 FORMAT(6X,F6.2,10X,E11.4,10X,E11.4,8X,E11.4,/,22X,E11.4,9X,E11.4)
    WRITE(6,611) YT
611 FORMAT(/,13X,'TABULAR VALUES FOR YT= ',F7.3,' READ IN')
    GO TO 9
12 ABPT=C.0
    ABNT=C.0
    ABST=C.0
3  IF(GRAIN.NE.2) GO TO 4
    ABPC=C.0
    ABNC=C.0
    ABSC=C.0
    GO TO 7
4  IF(Y.LE.0.0) READ(5,501) DC,DI,DELDI,S,THETAG,LGCI,LGNI,THETCN,THE
    ITCH
C *****
C *      READ IN BASIC GEOMETRY FOR C.P. GRAIN (NOT REQUIRED FOR      *
C *      STRAIGHT STAR GRAIN)                                         *
C *      DO IS THE AVERAGE OUTSIDE INITIAL GRAIN DIAMETER IN INCHES  *
C *      CI IS THE AVERAGE INITIAL INTERNAL GRAIN DIAMETER IN INCHES *
C *      DELDI IS THE DIFFERENCE BETWEEN THE INITIAL INTERNAL GRAIN  *
C *      DIAMETER AT THE NOZZLE END AND CI IN INCHES                 *

```

TABLE IV-2 (Cont'd)

```

C *      S IS THE NUMBER OF FLAT BURNING SLOT SIDES (NOT INCLUDING *
C *      THE NOZZLE END) *
C *      THETAG IS THE ANGLE THE NOZZLE END OF THE GRAIN MAKES WITH *
C *      THE MOTOR AXIS IN RADIAN *
C *      LGCI IS THE INITIAL TOTAL LENGTH OF THE CIRCULAR PERFORATION *
C *      IN INCHES *
C *      LGNI IS THE INITIAL SLANT LENGTH OF THE BURNING CONICAL *
C *      GRAIN AT THE NOZZLE END IN INCHES *
C *      THETCH IS THE CONTRACTION ANGLE OF THE BONDED GRAIN IN RAD. *
C *      THETCH IS THE CONTRACTION ANGLE AT THE HEAD END IN RADIAN *
C *****
501 FORMAT(5X,F7.3,6X,F7.3,9X,F7.3,5X,F4.0,9X,F7.5,/,7X,F7.2,7X,F6.2,9
1X,F7.5,9X,F7.5)
      IF(Y.LE.0.0) WRITE(6,601) DC,DI,CELDI,S,THETAG,LGCI,LGNI,THETCH,TH
      1ETCH
601 FORMAT(20X,'C.P. GRAIN GEOMETRY',/,13X,'DC= ',F7.3,/,13X,'DI= ',F7
1.3,/,13X,'CELDI= ',F7.3,/,13X,'S= ',F4.0,/,13X,'THETAG= ',F7.5,/,1
23X,'LGCI= ',F7.2,/,13X,'LGNI= ',F6.2,/,13X,'THETCH= ',F7.5,/,13X,'
3THETCH= ',F7.5,/)
      BNUM=ANUM*DC**2
      TLL=TL
      IF(ORDER.GE.3) TLL=C.0
      ABSC=S*ANUM*(DC**2-(DI+2.*Y)**2)
      IF(ABSC.LE.C.0) ABSC=C.0
      IF(2.*Y+DI.GT.DC) GO TO 100
      IF(THETAG.LE.C.08727) GO TO 101
      ABPC=3.14159*(DI+2.*Y)*((LGCI-(S+TAN(THETAG/2.))*Y-Y*COTAN(THETCH))-
1TLL)
      IF(ABPC.LE.C.0) ABPC=C.0
      ABNC=3.14159*(LGNI-Y*COTAN(THETAG+THETCH)-Y*TAN(THETAG/2.))*((DI+DE
1LDI+Y+LGNI*SIN(THETAG)+Y/SIN(THETAG+THETCH))*SIN(THETCH))
      IF(ABNC.LE.C.0) ABNC=C.0
      GO TO 5
100 ABNC=C.0
      ABPC=C.0
      GO TO 5
101 LGC=LGCI-Y*(COTAN(THETCH)+COTAN(THETCH))
      ABPC=3.14159*(DI+2.*Y)*(LGC-S*Y-TLL)
      IF(ABPC.LE.C.0) ABPC=C.0
      ABNC=C.0
5 APHT=ANUM*(DI+2.*RHT)**2
      IF(APHT.GE.BNUM) APHT=BNUM
      IF(K.LT.2) APHT1=APHT
      APNT=ANUM*(DI+CELDI+2.*RNT)**2
      IF(APNT.GE.BNUM) APNT=BNUM
      IF(GRAIN.NE.1) GO TO 7
      ABPS=C.0
      AESS=C.0

```

TABLE IV-2 (Cont'd)

```

      ABNS=C.0
      GO TO 50
      7 IF(Y.LE.0.0) READ(5,502) NS,LGSI,NP,RC,FILL,NN
C *****
C *      READ IN BASIC GEOMETRY FOR STAR GRAIN (NOT REQUIRED FOR *
C *      STRAIGHT C.P. GRAIN) *
C *      NS IS THE NUMBER OF FLAT BURNING SLOT SIDES (NOT INCLUDING *
C *      THE NOZZLE END) *
C *      LGSI IS THE INITIAL TOTAL LENGTH OF THE STAR SHAPED *
C *      PERFORATED GRAIN IN INCHES *
C *      NP IS THE NUMBER OF STAR POINTS *
C *      RC IS THE AVERAGE STAR GRAIN OUTSIDE RADIUS IN INCHES *
C *      FILL IS THE FILLET RADIUS IN INCHES *
C *      NN IS THE NUMBER OF STAR NOZZLE END BURNING SURFACES *
C *****
502 FORMAT(5X,F4.0,7X,F7.2,5X,F4.0,5X,F7.3,9X,F7.3,5X,F4.0)
      IF(Y.LE.0.0) WRITE(6,602) NS,LGSI,NP,RC,FILL,NN
602 FORMAT(15X,'BASIC STAR GEOMETRY',/,13X,'NS= ',F4.0,/,13X,'LGSI= ',
1F7.2,/,13X,'NP= ',F4.0,/,13X,'RC= ',F7.3,/,13X,'FILL= ',F7.3,/,13X
2,'NN= ',F4.0,/)
      FY=FILL+Y
      IF(STAR.EQ.1) GO TO 20
      IF(Y.LE.0.0) READ(5,503) RP,TAUS
C *****
C *      READ IN GEOMETRY FOR TRUNCATED STAR (NOT REQUIRED FOR *
C *      STANDARD STAR) *
C *      RP IS THE INITIAL RADIUS OF THE TRUNCATION IN INCHES *
C *      TAUS IS THE THICKNESS OF THE PROPELLANT WEB AT THE BOTTOM *
C *      OF THE SLOTS IN INCHES *
C *****
503 FORMAT(5X,F7.3,7X,F7.3)
      IF(Y.LE.0.0) WRITE(6,603) RP,TAUS
603 FORMAT(20X,'TRUNCATED STAR GEOMETRY',/,13X,'RP= ',F7.3,/,13X,'TAUS
1= ',F7.3,/)
      THETAS=3.14159/NP
      RPY=RP+Y
      LS=RC-TAUS-FILL-RP
      RPL=RP+LS
      THETS1=THETAS-ARSIN(FY/RPY)
      IF(THETS1.LE.0.0) GO TO 110
      IF(Y.LE.TAUS) GO TO 103
      THETAC=ARSIN((RC**2-RPL**2-FY**2)/(2.*FY*RPL))
      IF(THETAC.GE.0.0) GO TO 104
      IF(Y.LT.RC-RP) GO TO 105
      SG=0.0
      GO TO 14
103 SG=2.*NP*(RPY*THETS1+LS-(RPY*COS(THETAS-THETS1)-RP)+3.14159/2.*FY)
      GO TO 14

```

TABLE IV-2 (Cont'd)

```

104 SG=2.*NP*(RPY*THETS1+LS-(RPY*COS(THETAS-THETS1)-RP)+FY*THETAC)
    GO TO 14
105 SG=2.*NP*(RPY*THETS1+SQRT(RC**2-FY**2)-SQRT(RPY**2-FY**2))
    14 AGS=3.14159*(RC**2-RP**2)-NP*(3.14159*FILL**2/2.+2.*LS*FILL)
    GO TO 31
110 THETAF=THETAS
    THETAP=2.*THETAS
    TAUWS=TAUS
    GO TO 111
20 IF(Y.LE.0.0) READ(5,504) THETAF,THETAP,TAUWS
C *****
C *      READ IN GEOMETRY FOR STANDARD STAR (NCT REQUIRED FOR      *
C *      TRUNCATED STAR)                                           *
C *      THETAF IS THE ANGLE LOCATION OF THE FILLET CENTER IN RADIANS *
C *      THETAP IS THE ANGLE OF THE STAR POINT IN RADIANS          *
C *      TAUWS IS THE WEB THICKNESS OF THE GRAIN IN INCHES        *
C *****
504 FORMAT(9X,F7.5,9X,F7.5,8X,F6.3)
    IF(Y.LE.0.0) WRITE(6,604) THETAF,THETAP,TAUWS
604 FORMAT(20X,'STANDARD STAR GEOMETRY',/,13X,'THETAF= ',F7.5,/,13X,'T
    THETAP= ',F7.5,/,13X,'TAUWS= ',F6.3,/)
    THETAS=3.14159/NP
    THETS1=1.00
111 LF=RC-TAUWS-FILL
    CNUM=(Y+FILL)/LF
    DNUM=SIN(THETAF)/SIN(THETAP/2.)
    ENUM=(RC**2-LF**2-FY**2)/(2.*LF*FY)
    FNUM=SIN(THETAF)/COS(THETAP/2.)
    IF(CNUM.LE.FNUM) GO TO 106
    IF(Y.LE.TAUWS) GO TO 107
    SG=2.*NP*FY*(THETAF+ARSIN(SIN(THETAF)/CNUM)-ARCOS(ENUM))
    GO TO 23
106 IF(Y.LE.TAUWS) SG=2.*NP*LF*(DNUM+CNUM*(3.14159/2.+THETAS-THETAP/2.
    1-COTAN(THETAP/2.))+THETAS-THETAF)
    IF(Y.LE.TAUWS) GO TO 23
    SG=2.*NP*(FY*ARSIN(ENUM-(THETAS-THETAP/2.))+LF*DNUM-FY*COTAN(THETA
    1P/2.))
    GO TO 23
107 SG=2.*NP*LF*(CNUM*(THETAS+ARSIN(SIN(THETAF)/CNUM))+THETAS-THETAF)
    23 IF(THETS1.LE.0.0) GO TO 14
    AGS=3.14159*RC**2-NP*LF**2*(SIN(THETAF)*(COS(THETAF)-SIN(THETAF)*C
    1OTAN(THETAP/2.))+THETAS-THETAF+2.*FILL/LF*(SIN(THETAF)/SIN(THETAP/
    22.))+THETAS-THETAF+FILL/(2.*LF)*(3.14159/2.+THETAS-THETAP/2.-COTAN(
    3THETAP/2.)))
31 IF(SG.LE.0.0) SG=0.0
    IF(K.EQ.0) SGN=SG
    IF(K.LE.1) SGT=SG
    IF(K.EQ.2) SGN=SG

```

TABLE IV-2 (Cont'd)

```

IF(Y.LE.0.0) SG2=SG
IF(K.EQ.2) GO TO 37
RAVEDT=R1+(SG+SG2)/2.*RBAR*DELTAT
RNCT=R2+(SG+SG2)/2.*RRAVE*DELTAT
RHDT=R3+(SG+SG2)/2.*RRAVE*DELTAT
R1=RAVEDT
R2=RNCT
R3=RHDT
SG2=SG
GO TO 38
37 IF(KOUNT.NE.1) GO TO 39
SG3=SG
R4=R1
R5=R2
R6=R3
39 RAVEDT=R4+(SG+SG3)/2.*RBAR*DELTAT
RNCT=R5+(SG+SG3)/2.*RRAVE*DELTAT
RHDT=R6+(SG+SG3)/2.*RRAVE*DELTAT
R4=RAVEDT
R5=RNCT
R6=RHDT
SG3=SG
38 ABSS=(AGS-RAVEDT)*NS
IF(ABSS.LE.0.0) ABSS=0.0
IF(SG.LE.0.0) ABSS=0.0
ABNS=(AGS-RNCT)*NN
IF(ABNS.LE.0.0) ABNS=0.0
IF(SG.LE.0.0) ABNS=0.0
IF(ORDER.LE.2) ABPS=(LGSI-Y*(NS+NN))*SG
IF(ORDER.LE.2) GO TO 36
ABPS=(LGSI-TL-Y*(NS+NN))*SG
36 APHS=3.14159*RC**2-AGS+RHDT
IF(APHS.GE.3.14159*RC**2) APHS=3.14159*RC**2
IF(SG.LE.0.0) APHS=3.14159*RC**2
APNS=3.14159*RC**2-AGS+RNCT
IF(K.LT.2) APHS1=APHS
IF(APNS.GE.3.14159*RC**2) APNS=3.14159*RC**2
50 IF(NT.EQ.0.0) GO TO 52
IF(Y.LE.0.0) READ(5,506) LTP,DTP,THETTP,TAUEFF
C *****
C * READ IN GEOMETRY ASSOCIATED WITH TERMINATION PORTS (NOT *
* REQUIRED IF NT=0) *
* LTP IS THE INITIAL LENGTH OF THE TERMINATION PASSAGES *
* IN INCHES *
* DTP IS THE INITIAL DIAMETER OF THE TERMINATION PASSAGE *
* IN INCHES *
* THETTP IS THE ACUTE ANGLE BETWEEN THE AXIS OF THE PASSAGE *
* AND THE MOTOR AXIS IN RADIAN *

```

TABLE IV-2 (Cont'd)

```

C *      TAUEFF IS THE ESTIMATED EFFECTIVE WEB THICKNESS AT THE      *
C *      TERMINATION PORT IN INCHES                                  *
C *****
506 FORMAT(7X,F6.2,7X,F5.2,10X,F7.5,10X,F6.3)
    IF(Y.LE.0.0) WRITE(6,606) LTP,DTP,THETTP,TAUEFF
606 FORMAT(20X,'TERMINATION PORT GEOMETRY',/,13X,'LTP= ',F6.2,/,13X,'D
    LTP= ',F5.2,/,13X,'THETTP= ',F7.5,/,13X,'TAUEFF= ',F6.3,/)
    DABT=NT*3.14159*((DTP+2.*Y)*(LTP-Y/SIN(THETTP))-(DTP+2.*Y)**2/4.+
    1(Y+DTP/2.)*(DTP/2.)*(1.-1./SIN(THETTP)))
    IF(Y.GE.TAUEFF) DABT=0.0
52  BBP=0.0
    BBS=0.0
    BBN=0.0
    ABPORT=ABPT+ABPC+ABPS+DABT+BBP
    ABSLOT=ABST+ABSC+ABSS+BBS
    ABNOZ=ABNT+ABNC+ABNS+BBN
    SUMAB=ABPORT+ABSLOT+ABNOZ
    IF(K.EQ.0) GO TO 99
    IF(K.EQ.1) ABMAIN=ABPCRT+ABSLCT+ABNCZ
    K=K+1
    IF(K.GT.2) GO TO 69
    GO TO 2
69  ABTC=ABPORT+ABSLOT+ABNOZ
99  CONTINUE
    IF(Y.GT.0.0) GO TO 70
    ABP1=ABPCRT
    ABN1=ABNOZ
    ABS1=ABSLOT
70  ABP2=(ABP1+ABPCRT)/2.
    ABN2=(ABN1+ABNCZ)/2.
    ABS2=(ABS1+ABSLCT)/2.
    IF(INPUT.EQ.1) GO TO 76
    GO TO (71,72,73,74),ORDER
71  APHEAD=APHS1
    APNOZ=APNT
    SG=SGH
    GO TO 75
72  APHEAD=APHT1
    APNCZ=APNT
    SG=0.0
    IF(GRAIN.EQ.3) SG=(SGH+SGN)/2.
    GO TO 75
73  APHEAD=APHT1
    APNCZ=APNS
    SG=SGN
    GO TO 75
74  APHEAD=APHS1
    APNCZ=APNS

```


TABLE IV-2 (Cont'd)

```
SG=SGN
GC TO 75
76 APHEAD=APHT
  APNOZ=APNT
75 Y=YB
  DIFF=SUMAB-SUM2
  DADY=DIFF/DELY
  ABP1=ABPORT
  ABN1=ABNOZ
  ABS1=ABSLOT
  IF(ZW.GE.O.C) GC TO 77
  ABM1=ABMAIN
  ABMAIN=ABTO
  ABTO=ABM1
77 RETURN
  END
```

TABLE IV-2 (Cont'd)

SUBROUTINE CUTPUT

```

C *****
C * SUBROUTINE CUTPUT CALCULATES BASIC PERFORMANCE PARAMETERS *
C * AND PRINTS THEM OUT *
C * (WEIGHT CALCULATIONS ARE PERFORMED IN THE MAIN PROGRAM) *
C * T IS THE TIME IN SECS *
C * Y IS THE DISTANCE BURNED IN INCHES *
C * RNCZ IS THE NOZZLE END BURNING RATE IN INCHES/SEC *
C * RHEAD IS THE HEAD END BURNING RATE IN INCHES/SEC *
C * PCNOZ IS THE STAGNATION PRESSURE AT THE NOZZLE END IN PSIA *
C * PHEAD IS THE PRESSURE AT THE HEAD END OF THE GRAIN IN PSIA *
C * PTAR IS THE PORT TO THROAT AREA RATIO *
C * MNCZ IS THE MACH NUMBER AT THE NOZZLE END OF THE GRAIN *
C * PATM IS THE ATMOSPHERIC PRESSURE AT ALTITUDE IN PSIA *
C * SUMAB IS THE TOTAL BURNING AREA OF PROPELLANT IN IN**2 *
C * SG IS THE BURNING PERIMETER IN INCHES OF THE STAR SEGMENT *
C * (IF ANY) *
C * CFVAC IS THE VACUUM THRUST COEFFICIENT *
C * FVAC IS THE VACUUM THRUST IN LBS *
C * F IS THE THRUST IN LBS *
C * F IS THE THRUST IN LBS *
C *****
REAL MGEN,MDIS,MNOZ,MN1,JROCK,N,L,ME1,ME,ISP,ITOT,MU,MASS,ISPVAC
REAL M2,MDBAR,ISP2,ITVAC
COMMON/CONST1/ZH,AE,AT,THETA,ALFAN,G
COMMON/CONST2/CAPGAM,ME,BOT,ZETAF,TB,HB,GAM
COMMON/VARIA1/Y,T,DELY,DELTAT,PCNCZ,PHEAD,RNCZ,RHEAD,SUMAB,PHMAX
COMMON/VARIA3/ITOT,ITVAC,JROCK,ISP,ISPVAC,MDIS,MNOZ,SG,SUMMT
YB=Y
IF(Y.LE.0.0) M2=MDIS
MDBAR=(M2+MDIS)/2.
SUMMT=SUMMT+MDBAR*DELTAT
PTAR=1./JROCK
PRES=(1.+BOT/2.*ME*ME)**(-GAM/BOT)
ALT=HB*(T/TB)**(7./3.)
PATM=14.696/EXP(C.43103E-04*ALT)
CF=CAPGAM*SQRT(2.*GAM/BCT*(1.-PRES**((BOT/GAM)))+AE/AT*(PRES-PATM/P
1CNCZ)
CFVAC=CF+AE/AT*PATM/PCNCZ
F=ZETAF*CCS(THETA)*PCNOZ*AT*((1.+COS(ALFAN))/2.*CF+(1.-COS(ALFAN))
1/2.*AE/AT*(PRES-PATM/PCNOZ))
IF(F.LE.0.0) F=0.0
IF(Y.LE.0.0) F2=F
FBAR=(F+F2)/2.
FVAC=ZETAF*CCS(THETA)*PCNOZ*AT*((1.+CCS(ALFAN))/2.*CFVAC+(1.-CCS(A
1LFAN))/2.*AE/AT*PRES)
IF(Y.LE.0.0) FV2=FVAC
FVBAR=(FV2+FVAC)/2.

```

TABLE IV-2 (Cont'd)

```

ISP=F/(MDIS*G)
ISPVAC=FVAC/(MDIS*G)
ITGT=ITOT+FBAR*DELTAT
ITVAC=ITVAC+FVBAR*DELTAT
M2=MDIS
F2=F
FV2=FVAC
IF(PHEAD.GT.PHMAX) PHMAX=PHEAD
WRITE(6,1) T,YB,RNOZ,RHEAD,PONoz,PHEAD,PTAR,MNOZ,SUMAB,SG,PATM,CFV
1AC,FVAC,F
1 FORMAT(13X,'TIME= ',F5.1,12X,'Y= ',F6.3,/,13X,'RNOZ= ',1PE11.4,'
1RHEAD= ',1PE11.4,' PCNOZ= ',1PE11.4,' PHEAD= ',1PE11.4,/,13X,'PT
2AR= ',1PE11.4,' MNOZ= ',1PE11.4,' SUMAB= ',1PE11.4,' SG= ',
31PE11.4,/,13X,'PATM= ',1PE11.4,' CFVAC= ',1PE11.4,' FVAC= ',1PE
411.4,' F= ',1PE11.4,/)
RETURN
END

```

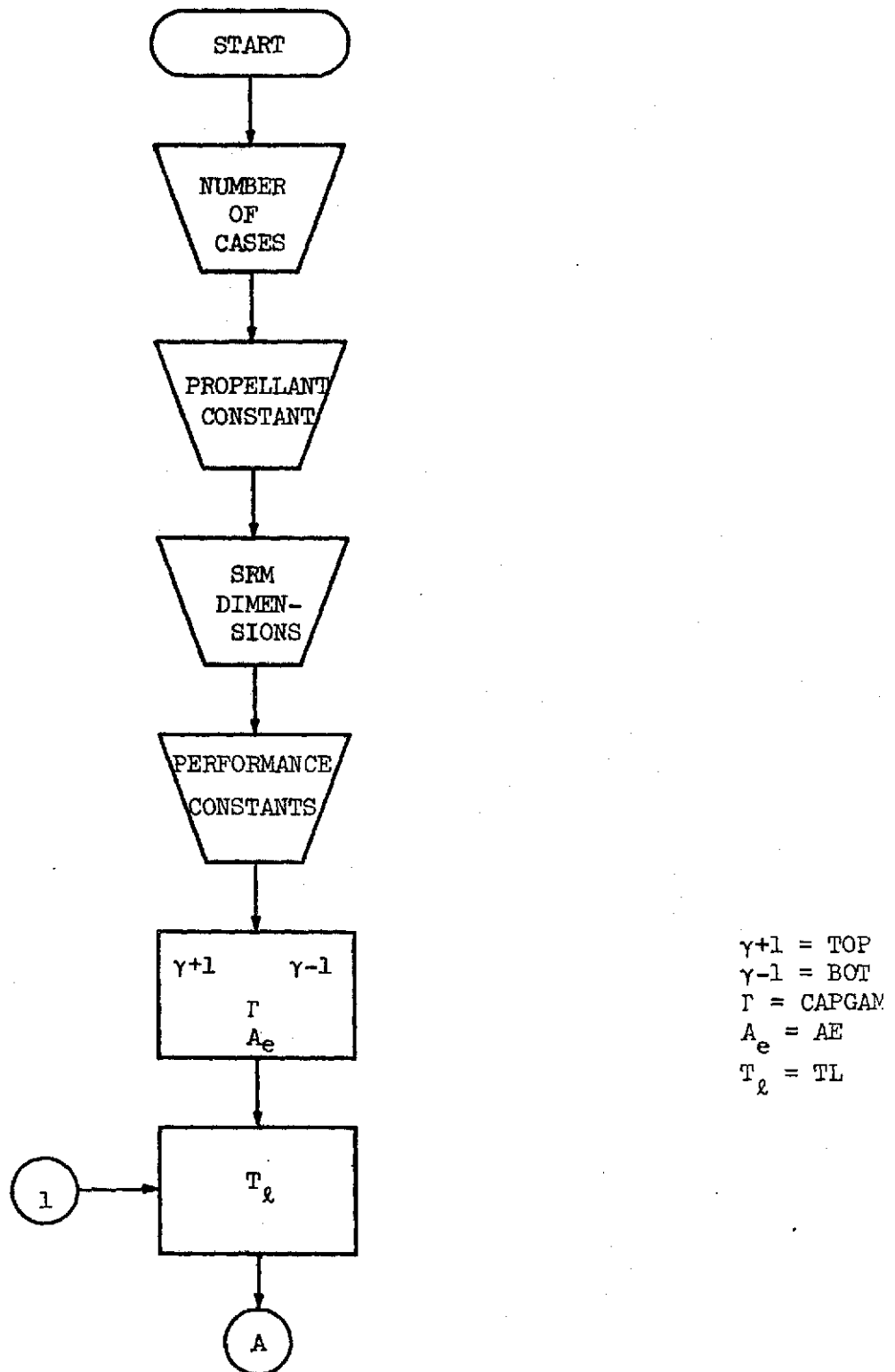


Figure IV-1. Flowchart for main program

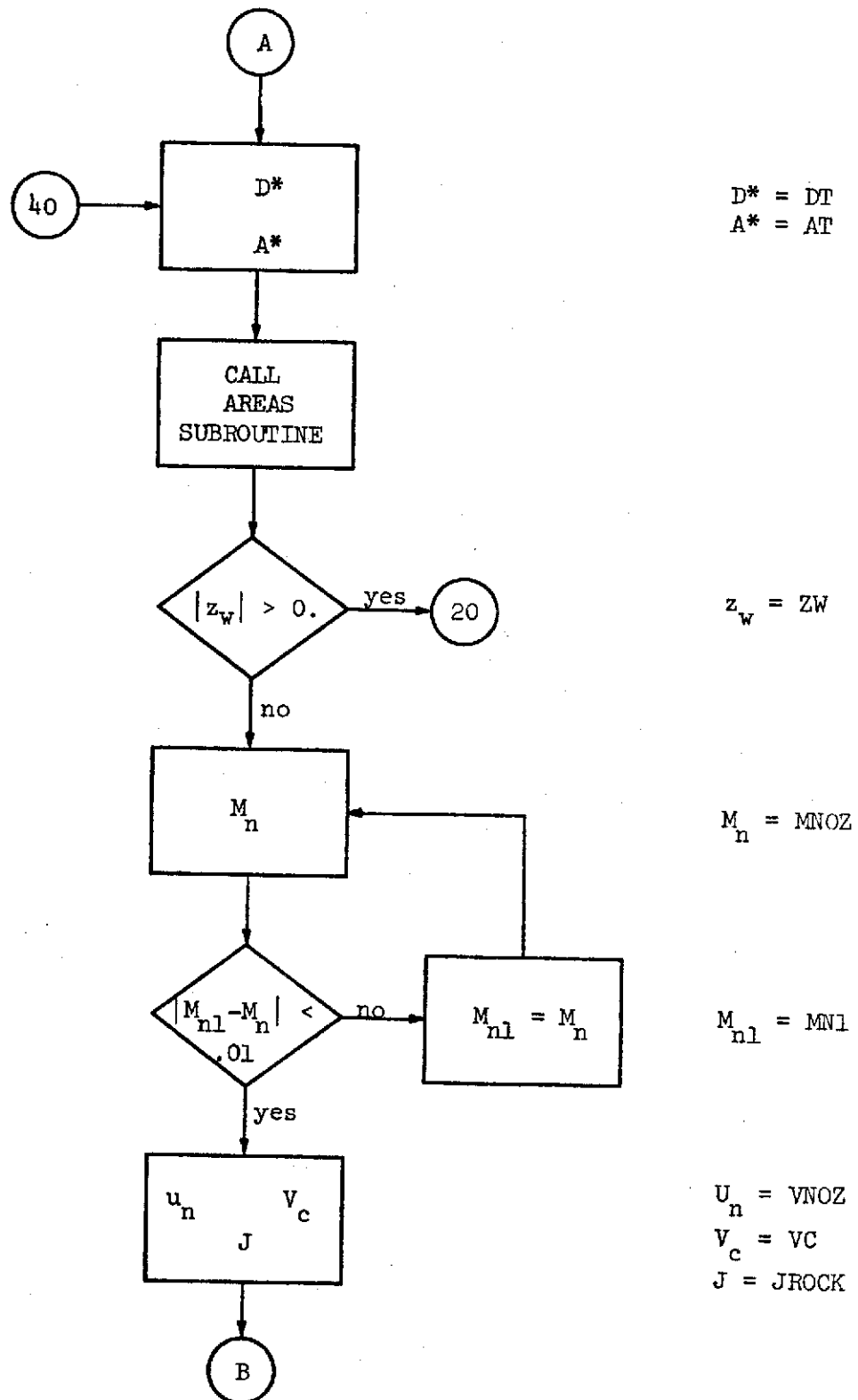


Figure IV-1 (Cont'd)

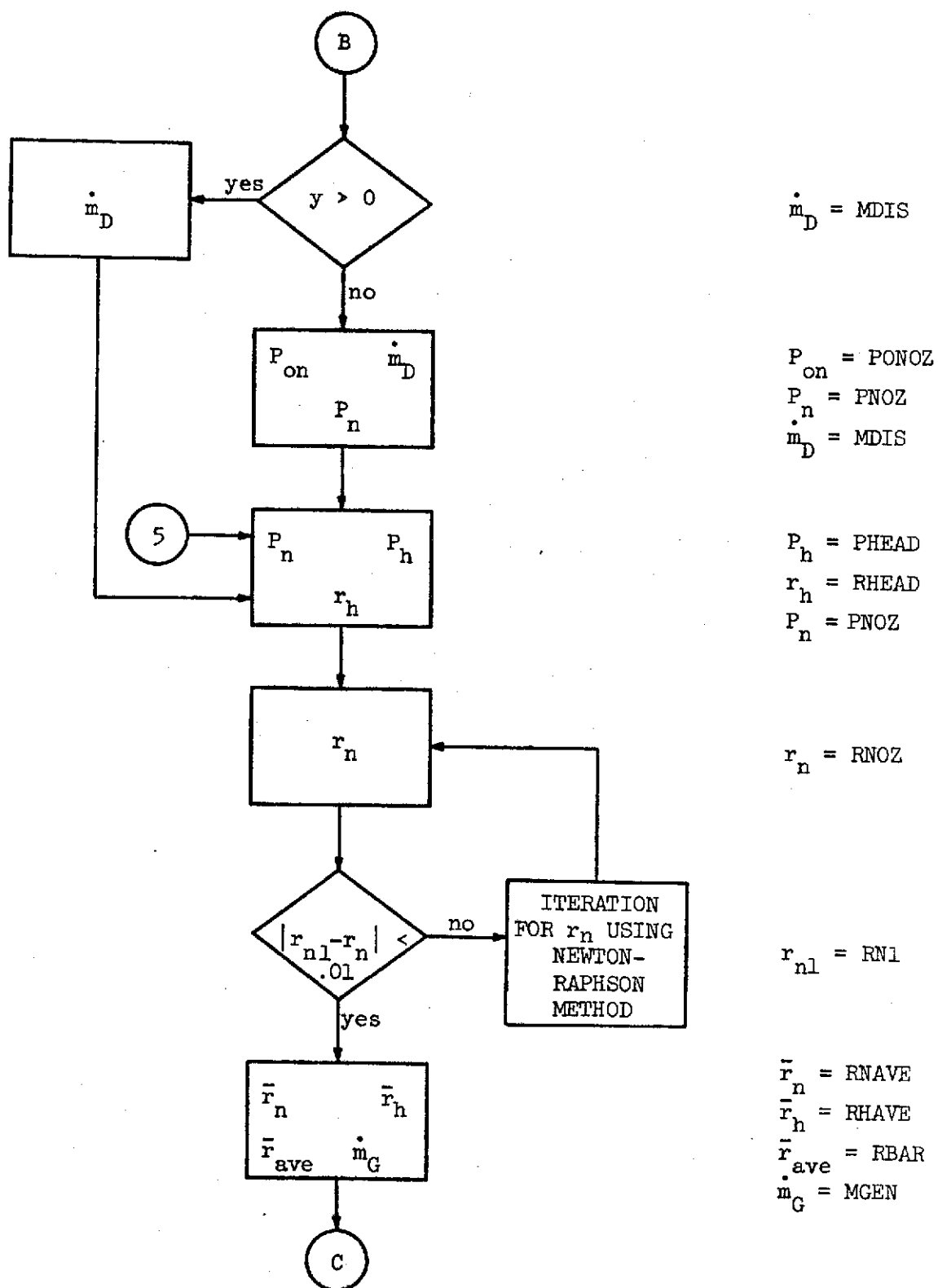


Figure IV-1 (Cont'd)

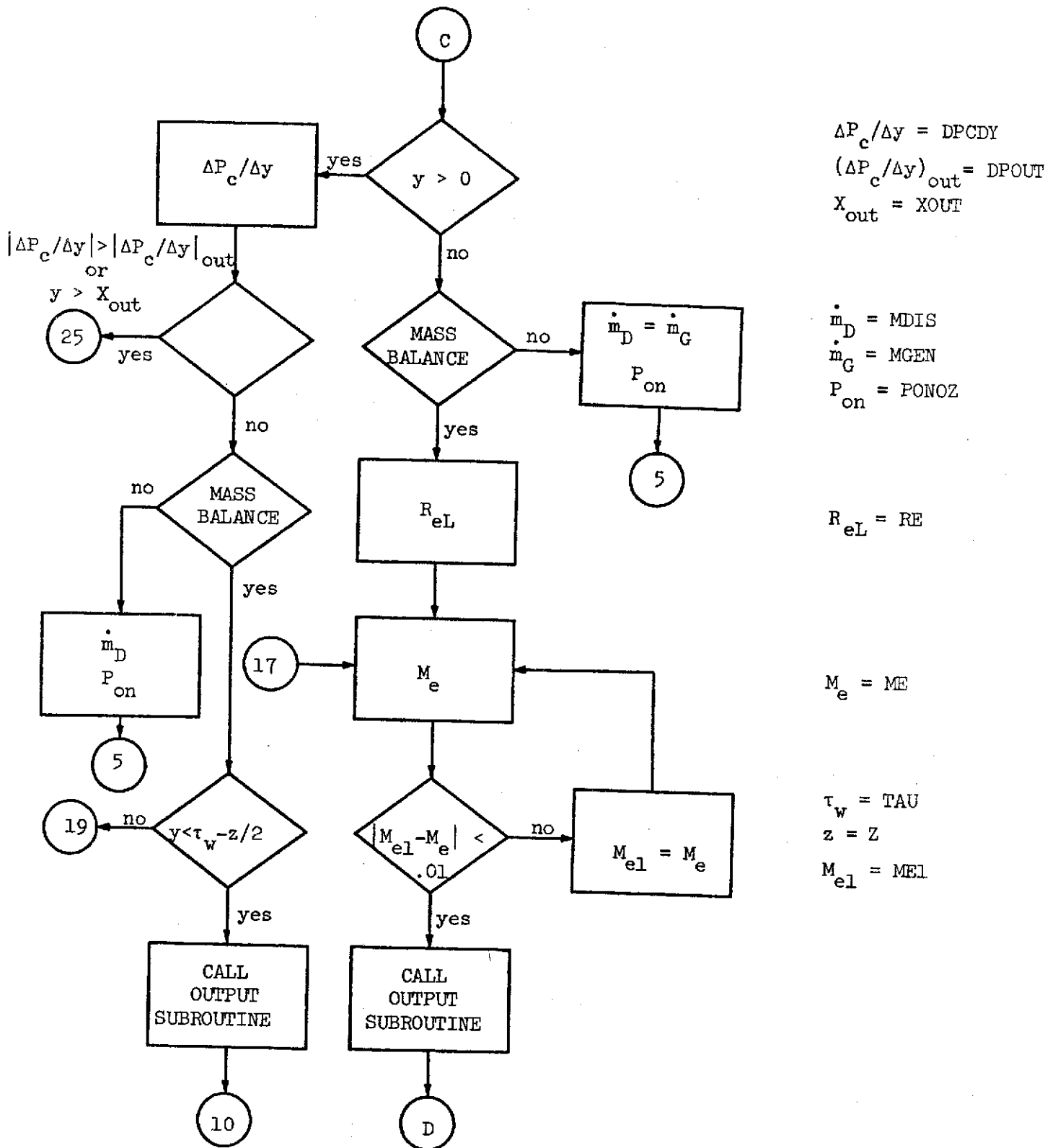


Figure IV-1 (Cont'd)

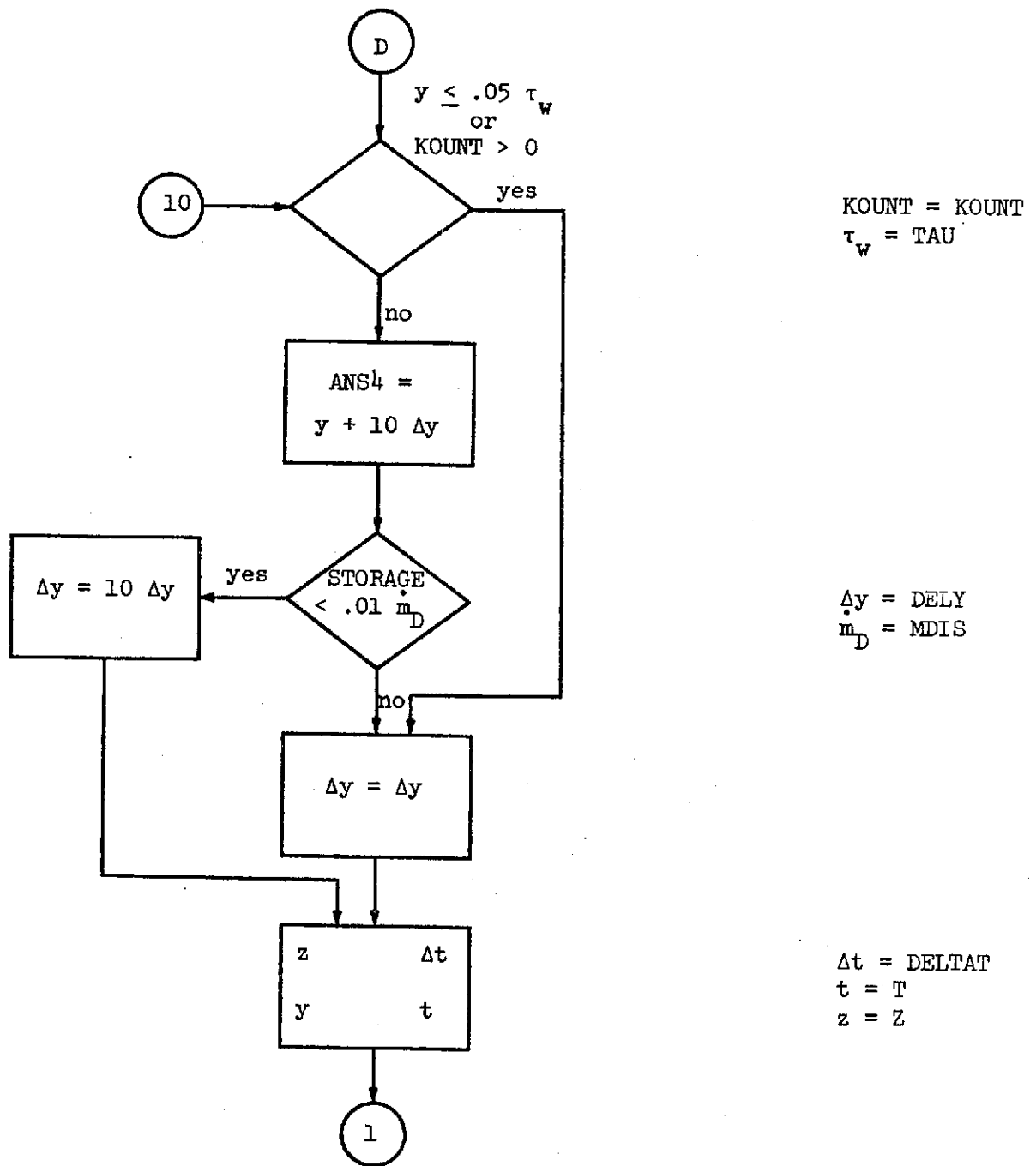


Figure IV-1 (Cont'd)

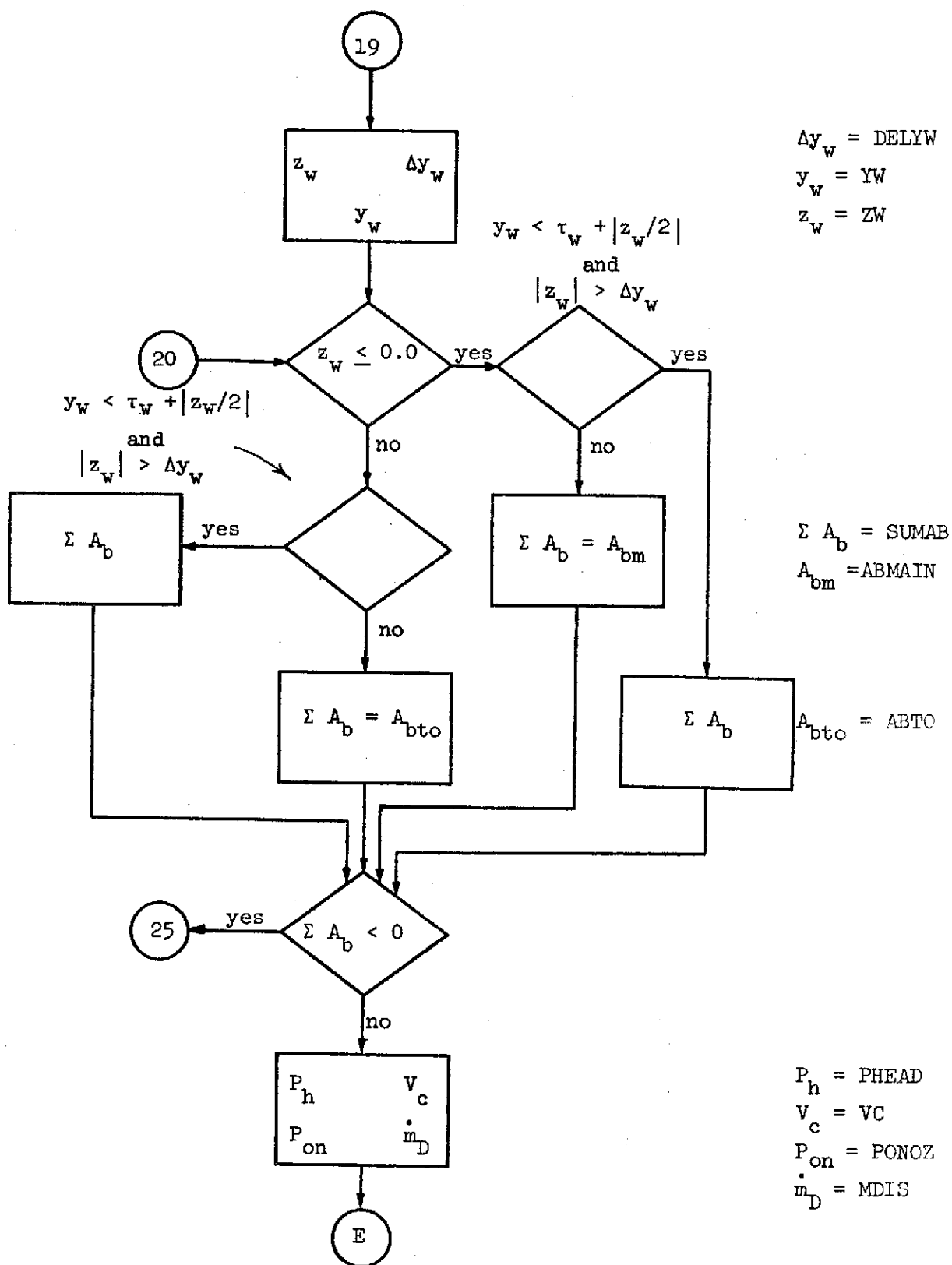


Figure IV-1 (Cont'd)

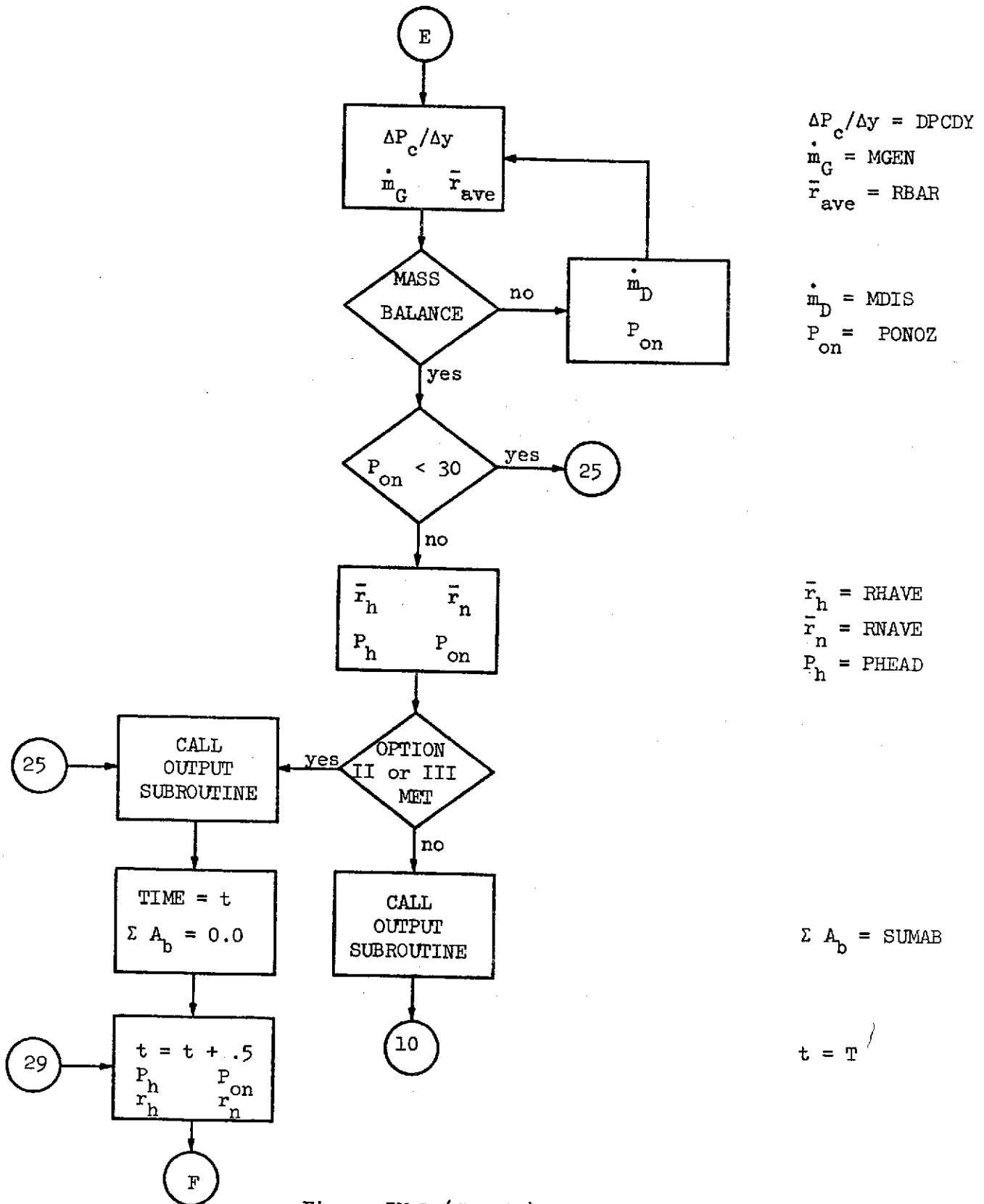
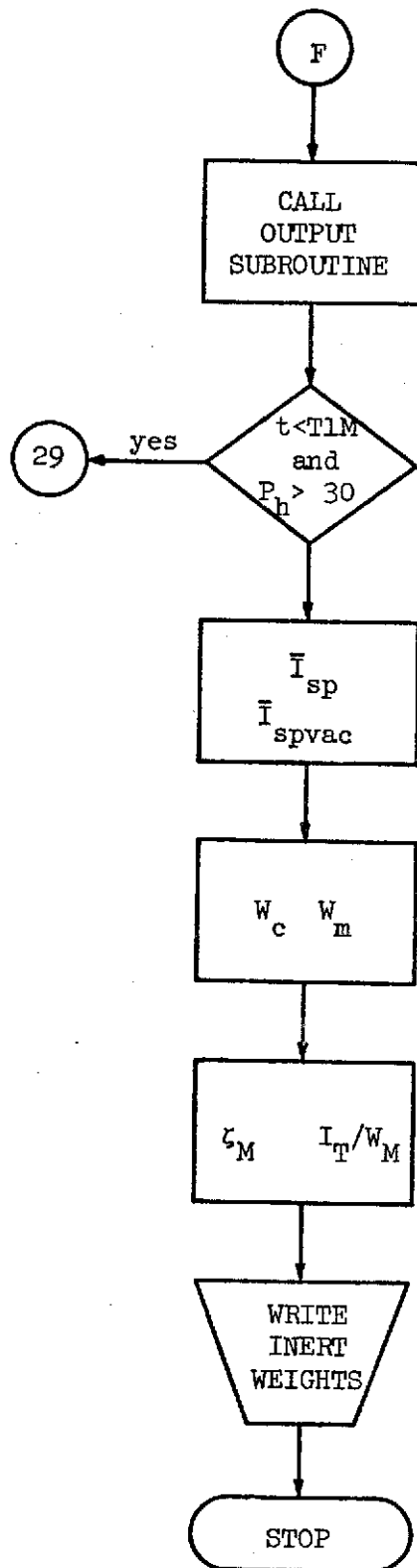


Figure IV-1 (Cont'd)



$\bar{I}_{sp} = ISP$
 $\bar{I}_{spvac} = ISPVAC$

$W_c = WC$
 $W_m = WM$

$\zeta_M = ZETAM$
 $I_T/W_M = RATIO$

Figure IV-1 (Cont'd)

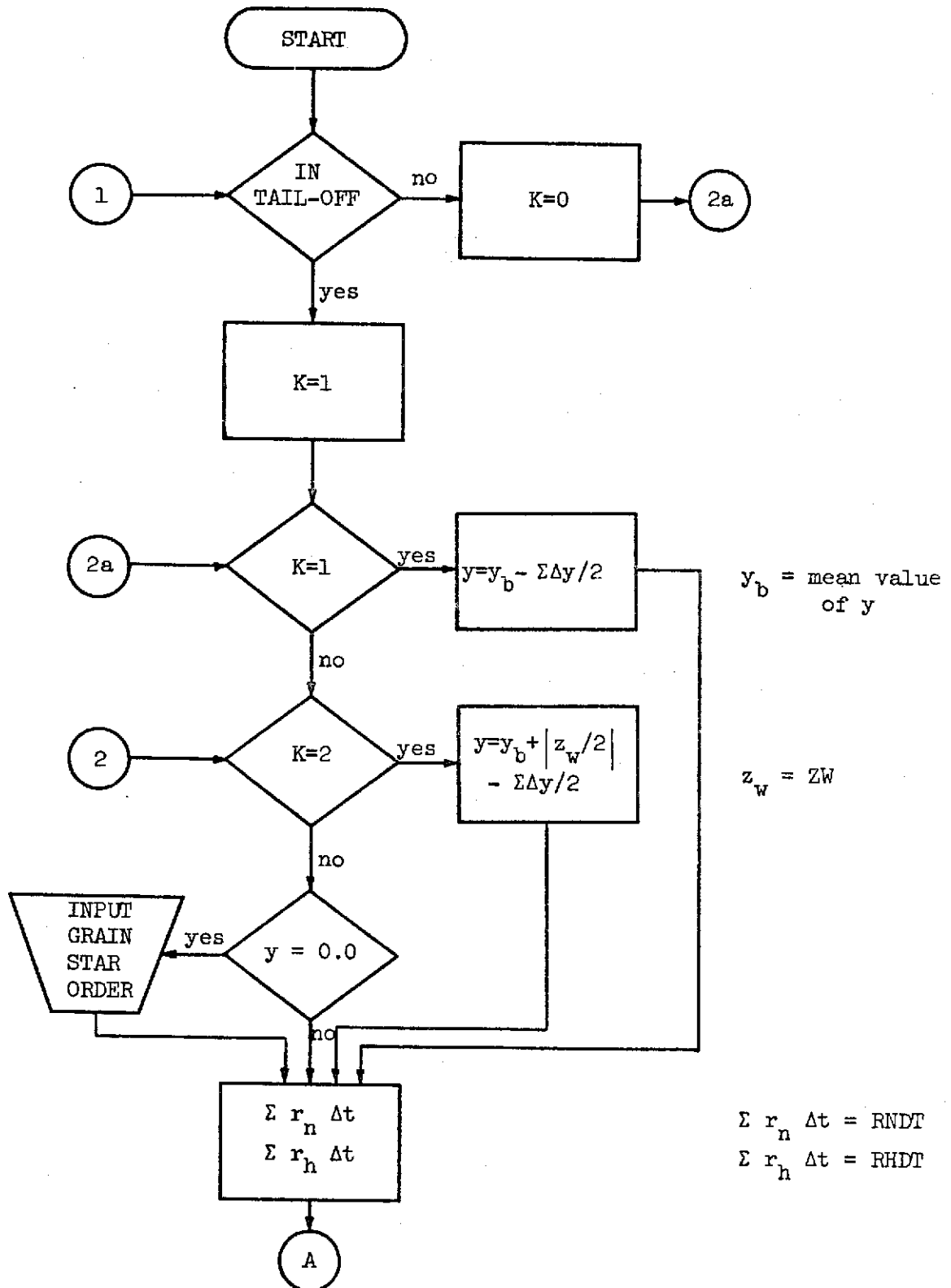
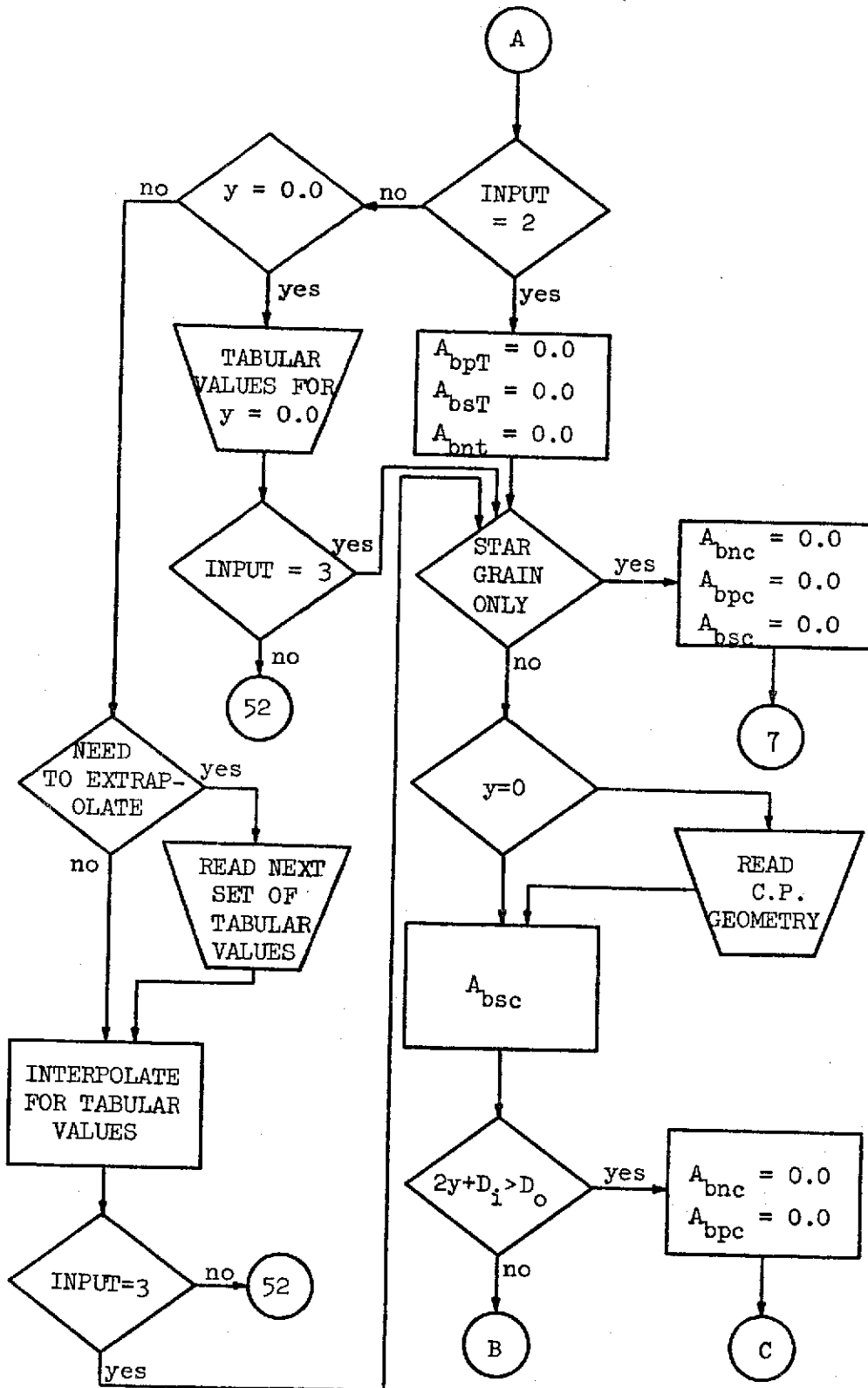


Figure IV-2. Flowchart for area subroutine



$A_{bpT} = ABPT$
 $A_{bsT} = ABST$
 $A_{bnT} = ABNT$

$A_{bnc} = ABNC$
 $A_{bpc} = ABPC$
 $A_{bsc} = ABSC$

$D_i = DI$
 $D_o = DO$

Figure IV-2 (Cont'd)

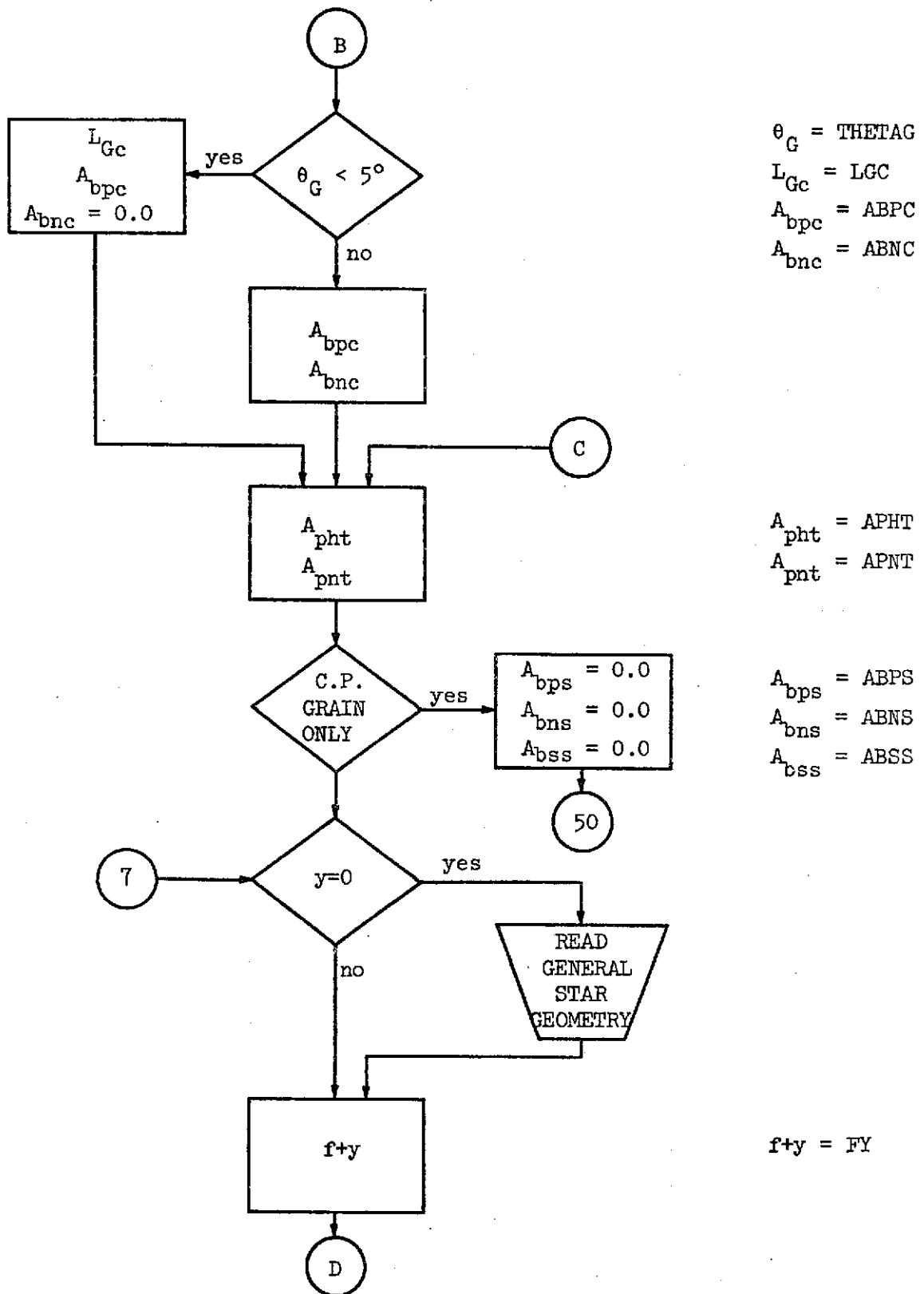


Figure IV-2 (Cont'd)

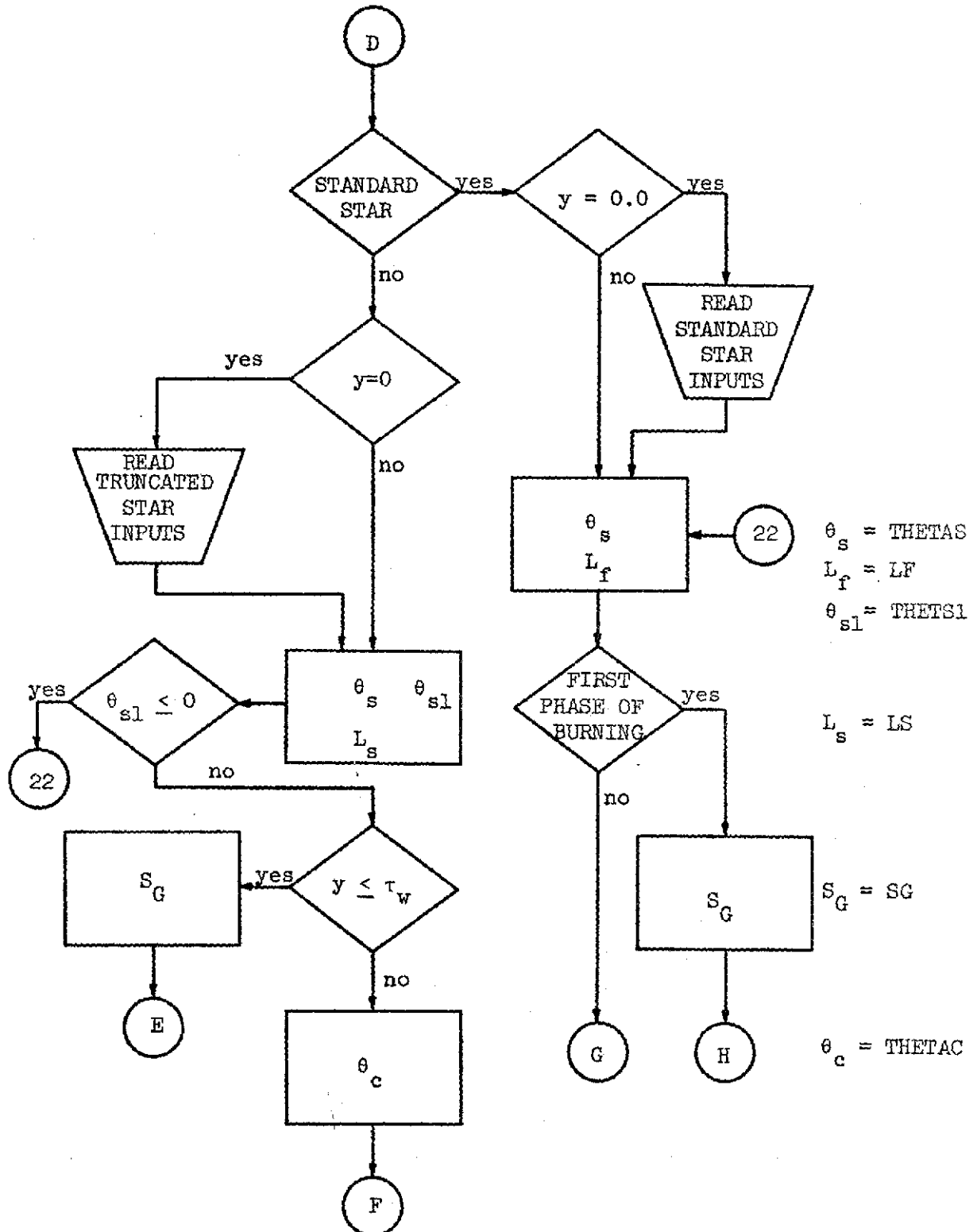


Figure IV-2 (Cont'd)

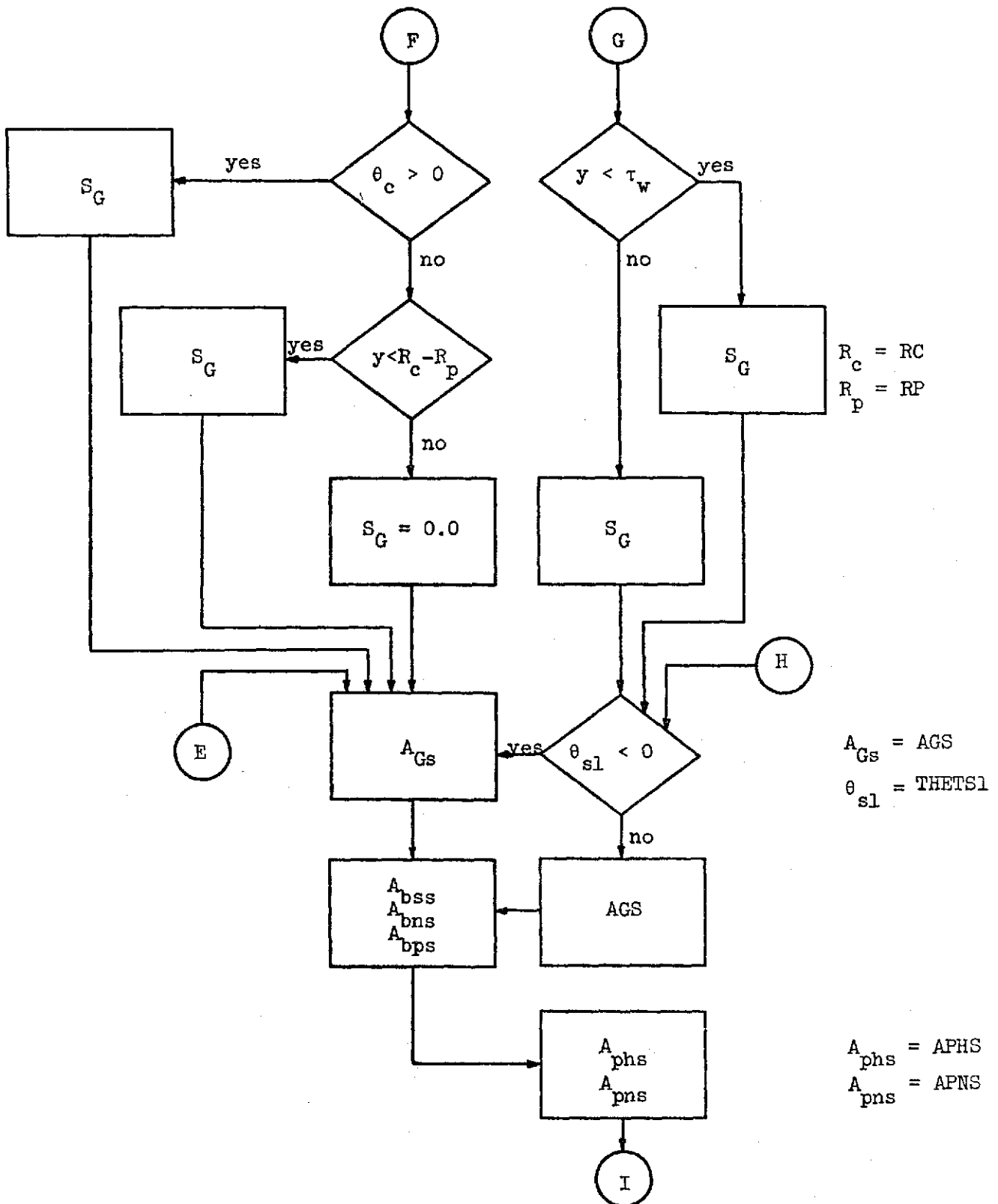
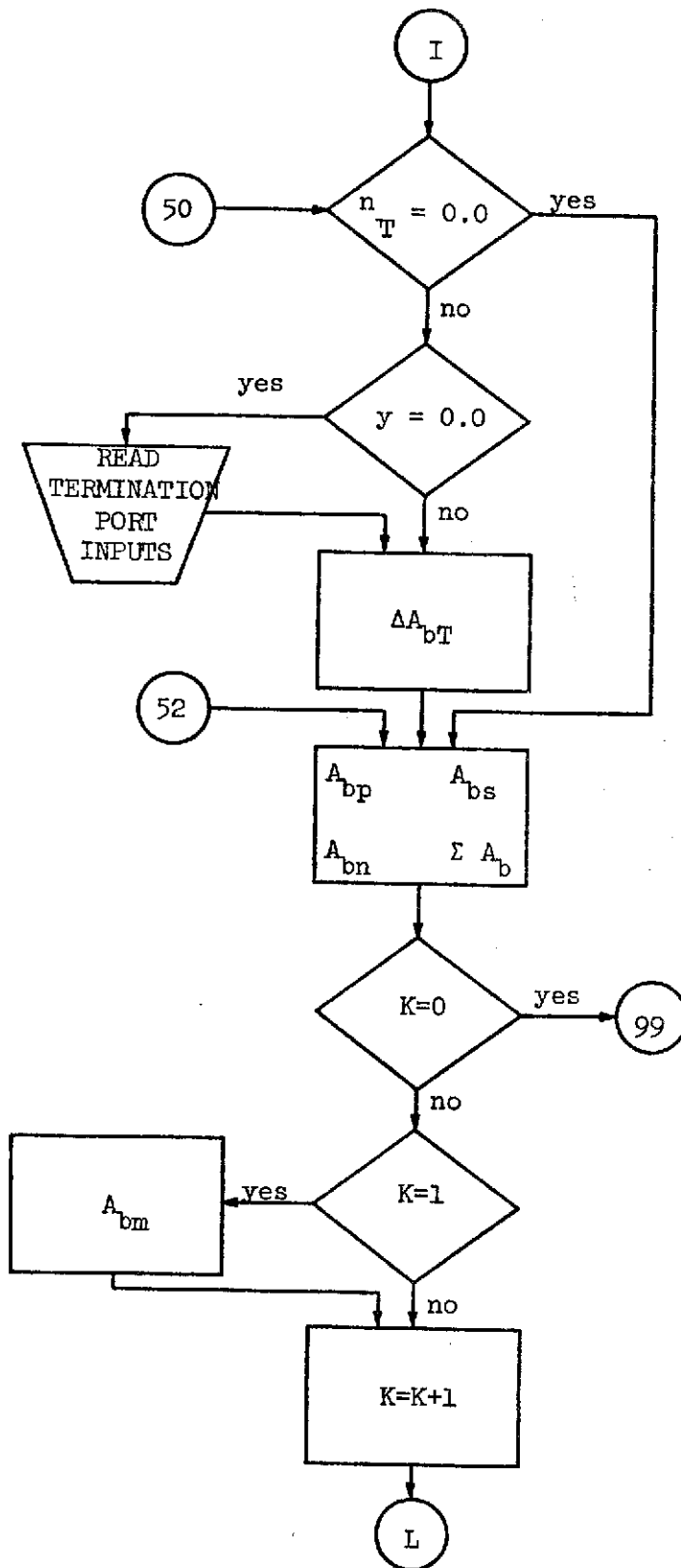


Figure IV-2 (Cont'd)



$n_T = NT$

$\Delta A_{bt} = DABT$

$A_{bp} = ABPORT$

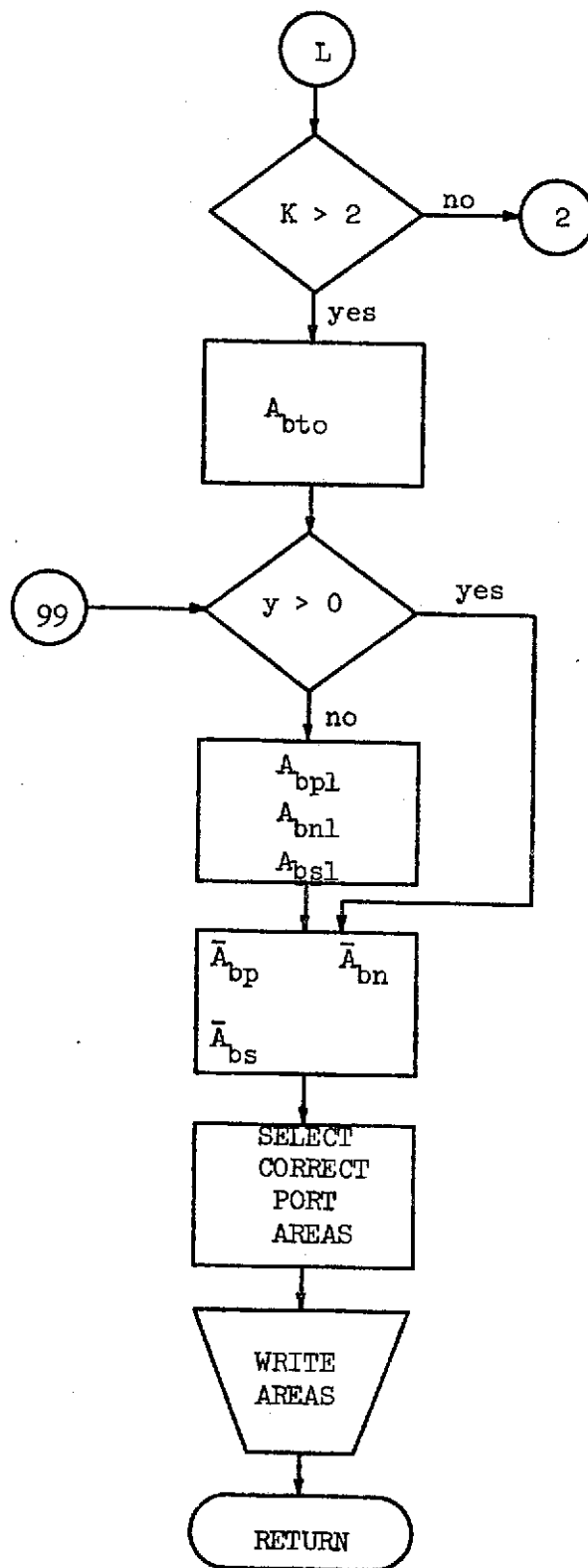
$A_{bs} = ABSLOT$

$A_{bn} = ABNOZ$

$\Sigma A_b = SUMAB$

$A_{bm} = ABMAIN$

Figure IV-2 (Cont'd)



$A_{bto} = ABTO$

$A_{bps} = ABPS$
 $A_{bnl} = ABN1$
 $A_{bsl} = ABS1$

$\bar{A}_{bp} = ABP2$
 $\bar{A}_{bs} = ABS2$
 $\bar{A}_{bn} = ABN2$

Figure IV-2 (Cont'd)

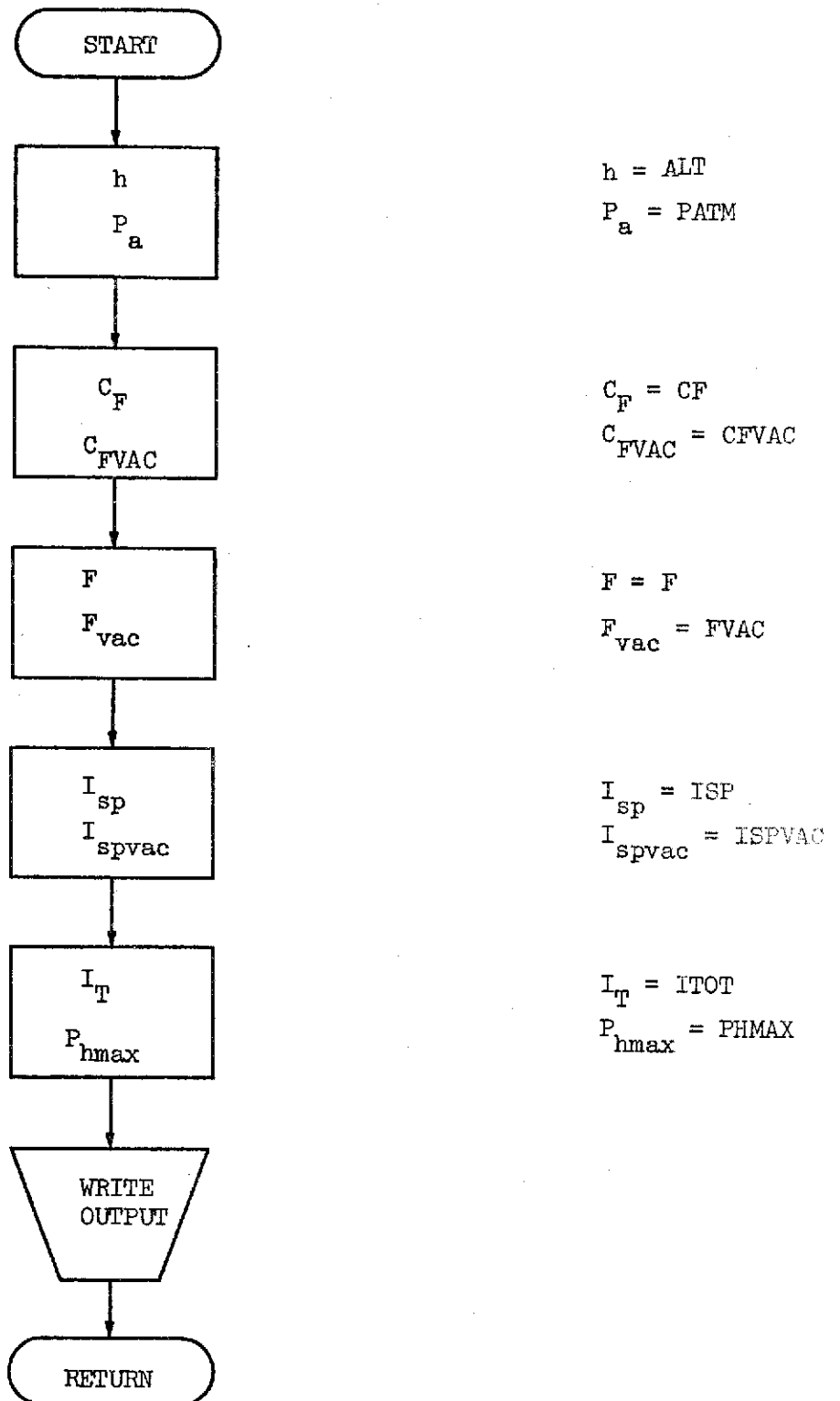


Figure IV-3. Flowchart for output subroutine

V. TEST CASES

In this section two test cases of the simplified computer program are presented. The complete input data and samples of the printout are tabulated. Plots of chamber pressure and thrust versus time are presented and compared with the results of others. Similar comparisons are made for weight calculations and related calculated values.

The first test case is for a 156 inch diameter SRM and the second for a 31 inch diameter SRM. For the latter, actual static test results are available for comparison. For the former, the comparison must be based on other theoretical results because the SRM has not been constructed.

Test Case 1 - SRM Number 1

SRM 1 is based on a design presented in Reference 4. The basic design is depicted on Figure V-1 and input data is tabulated in Table V-1. This design is of the general type under consideration for use by NASA as the Space Shuttle Booster. This particular SRM was selected because a relatively complete set of the required input data is presented in the reference as well as prediction results which are needed for comparison with the results of the simplified computer program. Also, the design is one which will test the use of the program for a combination circular perforated and star grain.

The SRM has three center segments consisting of circular perforated grains, an aft segment with a circular perforated grain and a forward segment with a truncated (slotted tube) star grain. Representation of the grain design is a straightforward process in this case. The aft dome must, however, be represented by a conical closure which is specified by selection of the angle θ_{cn} . Also, since a star grain is located in the forward dome an effective grain length must be specified and the star grain treated as being flat at both ends. Since this can introduce substantial error an adjustment is made by introduction of tabular input values. In this case the tabular inputs were based on a simple linear approximation to the effect of the curvature using as a guide the values of star grain perimeter obtained from a previous run in which the effect was neglected. The effect of the cutback in the grain inside diameter to accommodate the submerged nozzle was neglected in this case but the taper in the remaining portion of the grain, which substantially affects tailoff characteristics, was taken into account by specification of X_{Ta} and L_{Ta} .

The value of nozzle throat diameter used was 47.00 inches which differs from the 45.04 inches given in Reference 4. This substitution was made based on information received from NASA which indicated that the higher value had been used in predicting the performance data given in Reference 4. The program was tested with the erosion constants equal to zero because it is apparent that the predictions of Reference 4 were based on little or no propellant erosion.

A simple printout of the transients values for the initial portion of the trace computed by the program is given in Table V-2, and on Figure V-2 the chamber pressures and thrust versus time results are plotted and compared with results taken from Reference 4. The reference gives only one chamber pressure which is presumably the nozzle end stagnation pressure. It will be noted that the results for nozzle end stagnation pressure and thrust are in agreement within 5% except for the period between 110 to 125 seconds where the variation is somewhat greater. A large portion of the larger variation can be attributed to the approximations made in representing the effects of the closures. The remaining portion of the variation here as well as the small difference throughout the traces are more difficult to explain but are probably attributable to differences in analytical technique although a possibility exists that there could be some differences in the input data used.

It is notable that there is good agreement in the web time calculated by the several approaches. Weight and related calculated values are tabulated as Table V-3. Comparison with corresponding data from Reference 4 are made later in this section.

Test Case 2 - SRM Number 2

SRM 2 is based on design of the TX354-5 rocket motor as presented in Reference 9. The basic design is depicted on Figure V-3 and input data is tabulated in Table V-4. This SRM was selected because of the availability of a relatively complete set of input data as well as specific test results.

The grain has, in general, a cylindrical port with two radial slots. The main propellant grain geometry was represented by the circular perforated grain equations. The effect of the forward closure was represented by selection of θ_{ch} . Radii at the slots were neglected except at the aft face of the aft slot. Here the geometry of the entire portion of the propellant aft of the aft slot was represented by tabular input values because the step change in port area does not permit accurate representation through the equation inputs. In specification of x_{Ta} the effect of the tapered insulation thickness at the bottom of the aft slot was taken into account so that x_{Ta} was taken somewhat larger than it would have been based on the inside taper alone.

As data on the erosive burning constants of the propellant was not available, arbitrary values were devised to give a moderate erosion rate. These values are listed under ALPHA and BETA in Table V-4. The tests were also run with ALPHA and BETA both assigned zero values; i.e., no erosive burning. A sample printout of the transients values calculated for the final portion of the trace is given in Table V-5. These are for finite values of ALPHA and BETA. On Figure V-4 the head end pressure and thrust versus time results are plotted and compared with static test results reported in Reference 9. The chamber pressure results of the typical test given in the reference are in this case presumably head end pressure since this is the pressure routinely recorded in standard test firings.

Comparison of the program output with the test data of Reference 9 shows that both sets of assumed values for erosive burning constants give only fair representation of test data for the initial portion of the trace. (It should be noted that the initial peaks in the test results may be at least partially attributable to the ignitor, effects of which are not considered in the program). The agreement for the remaining portion of the trace is again within 5% except for a small portion of the trace shortly before tail-off. The larger variation near tailoff may be traced directly to the approximation of the effect of the forward closure made by the specification of θ_{ch} . Here an adjustment could be easily introduced to correct precisely or approximately for the effect through use of the tabular input values. This was not done in the test case so that the result of the approximation could be evaluated by the reader.

Again there is good agreement in the web time calculated by the program and that given by the reference. The agreement is poorer in case of action time probably due principally to neglect of the effect of chamber wall heating on chamber depressurization in the simplified program.

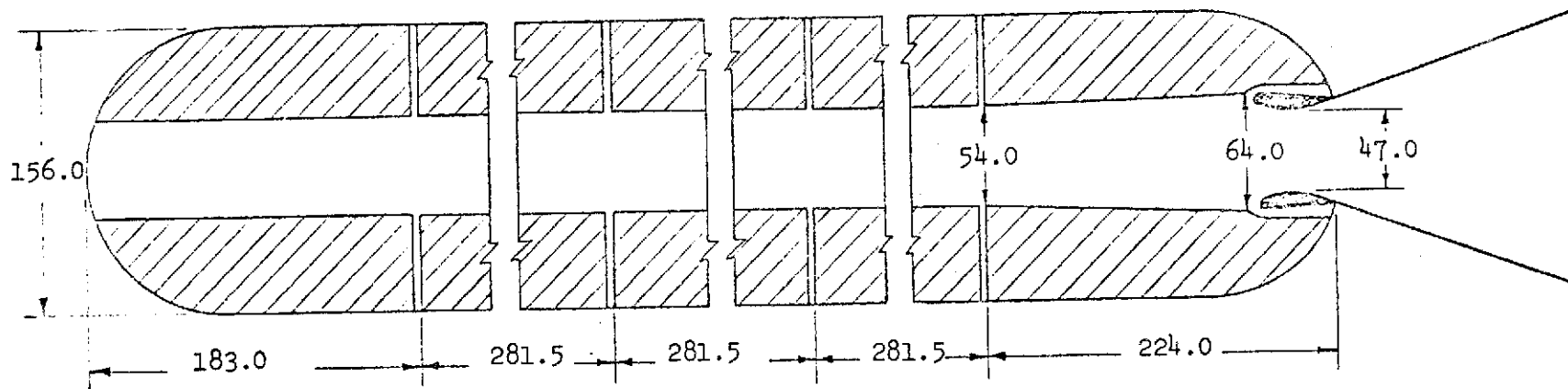
Comparisons of Weights and Related Values

Weights and related values for both test cases are tabulated in Table V-3. The values for SRM 2 are based on the finite erosion rates.

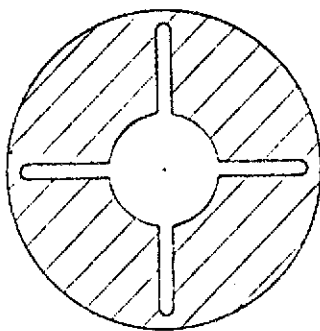
Generally excellent agreement is found between the program and the reference results for propellant weights and impulse values for both SRMs. For SRM 1, a substantial difference in the value of P_{meop} exists which results in rather large differences in the case thicknesses, case weights and nozzle weights. It is believed that this may be principally the results of differences in the ballistic variability factors used in the two predictions. The rather large difference in insulation weights for SRM 1 is attributed to neglect of requirements for final thermal protection in the present program. (See step 64, Section III and W_a , Section II).

For SRM 2 substantial differences exist in the case and nozzle weights resulting in a similar difference in total motor weight. The difference in case weight is attributable to two factors: 1) the use of approximately 10 percent additional basic chamber thickness in the actual motor to allow for manufacturing variations, and 2) the use of a thick wall case segment (0.25 inches in thickness) to accommodate a thrust ball bracket and related hardware. The difference in nozzle weight is principally the result of the assumption for the present program of a nozzle design radically different from that used in the TX354-5. (See K_1 and K_2 , Section II).

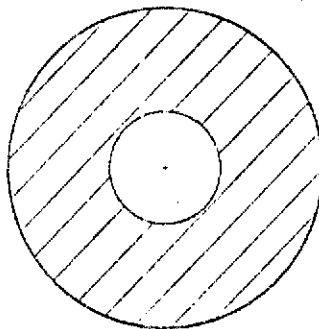
Additional testing of the program was accomplished in the process of compiling the design performance data given in the Appendix.



FORWARD SEGMENT



CENTER SEGMENTS



AFT SEGMENT

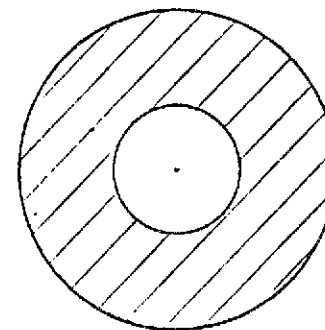


Figure V-1. Sketch of SRM Number 1 (Reference 4)
All dimensions are in inches.

TABLE V-1. INPUT DATA FOR SRM NO. 1

PROPELLANT CHARACTERISTICS		
RHO= 0.001990		
A= 0.6712		
N= 0.250	BASIC STAR GEOMETRY	
ALPHA= 0.0	NS= 1.	
BETA= 0.0	LGS1= 148.00	
MU= 1.2700E-07	NP= 4.	
CSTAR= 5.1860E 03	RC= 77.470	
	FILL= 2.500	
	NN= 0.	
BASIC MOTOR DIMENSIONS		
L= 1243.		
TAU= 50.47	TRUNCATED STAR GEOMETRY	
VCI= 3.2300E 06	RP= 27.000	
VCF= 2.2400E 07	TAUS= 9.670	
DE= 1.4800E 02		
DII= 4.7000E 01		
THETA= 0.0		
ALFAN= 3.0340E-01		
LTAP= 1.5000E 02	INERT WEIGHT INPUTS	
XT= 5.0000E 00	PIPK= 1.5000E-03	
Z0= 0.0	DTEMP= 3.0000E 01	
	SIGMAP= 3.0000E-02	
	SIGMAS= 2.0000E-02	
	N1= 3.0000E 00	
BASIC PERFORMANCE CONSTANTS		
DELTA= 0.250	N2= 0.0	
XOUT= 1000.00	SYCNOM= 1.9000E 05	
DPOUT= 500.00	DCC= 1.5550E 02	
ZETA= 0.980	PSIC= 1.4000E 00	
TB= 130.0	DELC= 2.8300E-01	
HB= 150000.	LCC= 1.1025E 03	
GAM= 1.143	NSEG= 5.0000E 00	
RADER= 0.0135	HCN= 6.2000E 01	
	SYNNOM= 1.9000E 05	
	PSIS= 1.4000E 00	
	PSIA= 2.0000E 00	
GRAIN CONFIGURATION		
INPUT= 3	K1= 2.0800E-01	
GRAIN= 3	K2= 9.2500E-02	
STAR= 2	PSIINS= 2.0000E 00	
NT= 0.	DELINS= 4.6200E-02	
ORDER= 1	KEH= 3.0000E-03	
	KEN= 1.2000E-02	
	DLINER= 2.3000E-02	
	TAUL= 6.5000E-02	
	WA= 0.0	
C.P. GRAIN GEOMETRY		
DO= 154.940	TABULAR VALUES	
DI= 54.000		
DELDI= 0.0	Y	ABPK
S= 7.		
THETAG= 0.0	0.0	9,000
LCCI= 1048.50	9.67	9,000
LCNI= 0.0	40.0	0
THETCN= 0.75035	160.0	0
THETCH= 1.57075	All other A's = 0	

TABLE V-2. SAMPLE PRINTOUT OF TRANSIENT VALUES FOR SRM NO. 1

INITIAL REYNOLDS NUMBER = 1.1206E 09							
TIME= 0.0		Y= 0.0					
RNOZ=	3.9230E-01	RHEAD=	4.0029E-01	PON0Z=	9.2164E 02	PHEAD=	9.9906E 02
PTAR=	1.3201E 00	MNOZ=	5.2199E-01	SUMAB=	3.9086E 05	SG=	4.8836E 02
PATM=	1.4696E 01	CFVAC=	1.7775E 00	FVAC=	2.7270E 06	F=	2.4792E 06
TABULAR VALUES FOR YT= 9.670 READ IN							
TIME= 0.6		Y= 0.250					
RNOZ=	3.9198E-01	RHEAD=	3.9945E-01	PON0Z=	9.1861E 02	PHEAD=	9.9066E 02
PTAR=	1.3434E 00	MNOZ=	5.0941E-01	SUMAB=	3.9175E 05	SG=	4.8925E 02
PATM=	1.4696E 01	CFVAC=	1.7774E 00	FVAC=	2.7198E 06	F=	2.4721E 06
TIME= 1.3 Y= 0.500							
RNOZ=	3.9198E-01	RHEAD=	3.9898E-01	PON0Z=	9.1861E 02	PHEAD=	9.8601E 02
PTAR=	1.3669E 00	MNOZ=	4.9745E-01	SUMAB=	3.9263E 05	SG=	4.9015E 02
PATM=	1.4694E 01	CFVAC=	1.7773E 00	FVAC=	2.7216E 06	F=	2.4739E 06
TIME= 1.9 Y= 0.750							
RNOZ=	3.9198E-01	RHEAD=	3.9855E-01	PON0Z=	9.1861E 02	PHEAD=	9.8176E 02
PTAR=	1.3906E 00	MNOZ=	4.8606E-01	SUMAB=	3.9351E 05	SG=	4.9107E 02
PATM=	1.4691E 01	CFVAC=	1.7772E 00	FVAC=	2.7235E 06	F=	2.4758E 06
TIME= 2.5 Y= 1.000							
RNOZ=	3.9218E-01	RHEAD=	3.9836E-01	PON0Z=	9.2053E 02	PHEAD=	9.7992E 02
PTAR=	1.4146E 00	MNOZ=	4.7520E-01	SUMAB=	3.9437E 05	SG=	4.9199E 02
PATM=	1.4686E 01	CFVAC=	1.7771E 00	FVAC=	2.7310E 06	F=	2.4834E 06
TIME= 3.2 Y= 1.250							
RNOZ=	3.9218E-01	RHEAD=	3.9799E-01	PON0Z=	9.2053E 02	PHEAD=	9.7631E 02
PTAR=	1.4387E 00	MNOZ=	4.6526E-01	SUMAB=	3.9523E 05	SG=	4.9293E 02
PATM=	1.4680E 01	CFVAC=	1.7770E 00	FVAC=	2.7328E 06	F=	2.4853E 06

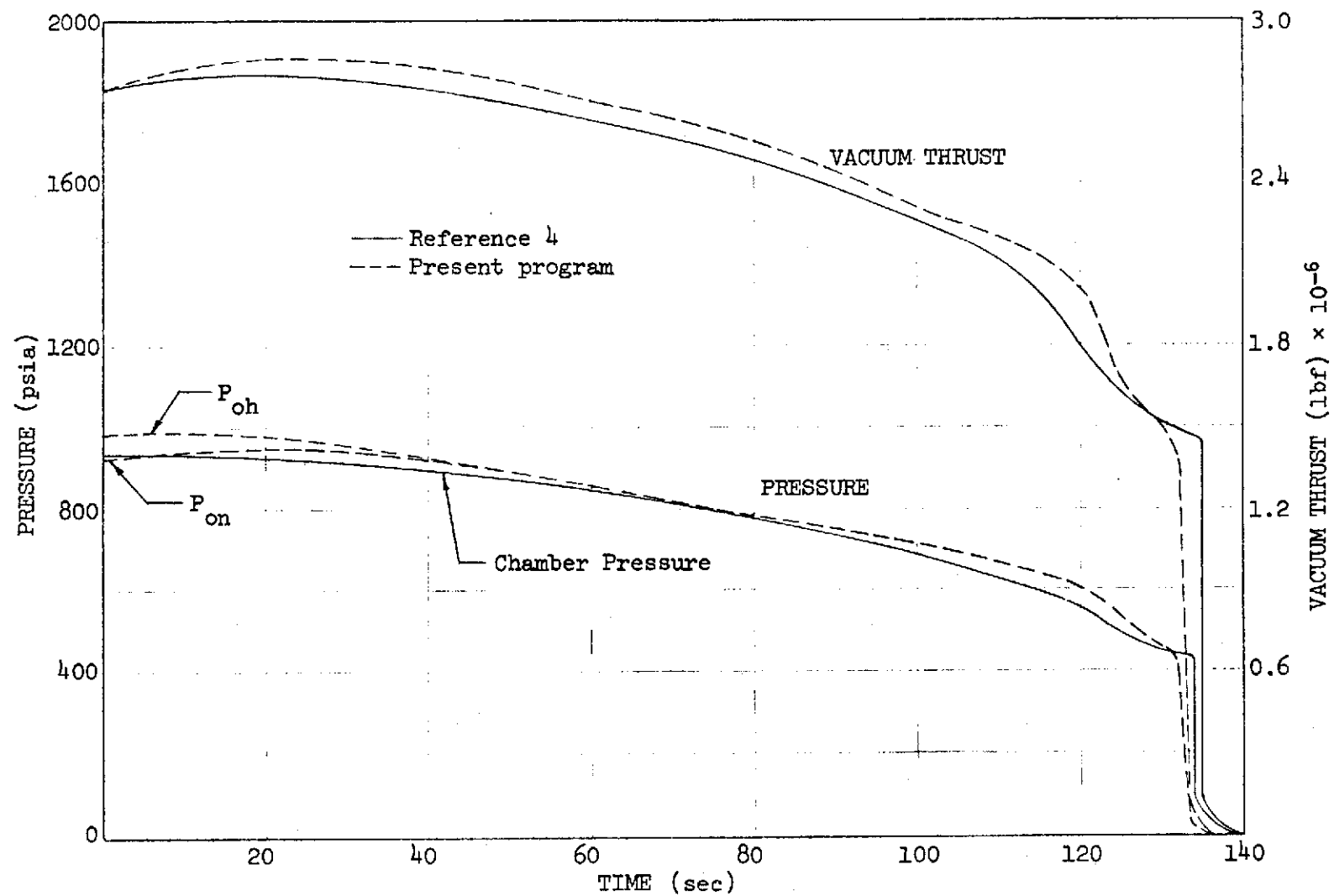


Figure V-2. Pressures and vacuum thrust versus time for SRM number 1.

TABLE V-3. PRINTOUT OF WEIGHTS AND RELATED VALUES

SRM Number 1

WP1= 1.2269E 06	
WP2= 1.2284E 06	
WP= 1.2276E 06	
PHMAX= 9.9906E 02	
ISP= 2.6215E 02	MAX EXPECTED PRESSURE= 1.1391E 03
ISPVAC= 2.7260E 02	CYLINDRICAL CASE THICKNESS= 6.5258E-01
ITOT= 3.2183E 08	CASE WT= 1.2395E 05
ITVAC= 3.2465E 08	NOZZLE WT= 1.2586E 04
	INSULATION WT= 9.3402E 03
	LINER WT= 1.3017E 03
	TOTAL MOTOR WT= 1.3748E 06
	ZETAM= 8.9295E-01
	RATIO OF ITOT TO WM= 2.3409E 02

SRM Number 2

WP1= 8.2173E 03	
WP2= 8.1896E 03	
WP= 8.2035E 03	
PHMAX= 7.2842E 02	
ISP= 2.3558E 02	MAX EXPECTED PRESSURE= 8.1058E 02
ISPVAC= 2.6223E 02	CYLINDRICAL CASE THICKNESS= 1.0019E-01
ITOT= 1.9326E 06	CASE WT= 5.7301E 02
ITVAC= 2.1512E 06	NOZZLE WT= 1.1390E 02
	INSULATION WT= 1.0105E 02
	LINER WT= 4.2155E 01
	TOTAL MOTOR WT= 9.0336E 03
	ZETAM= 9.0811E-01
	RATIO OF ITOT TO WM= 2.1394E 02

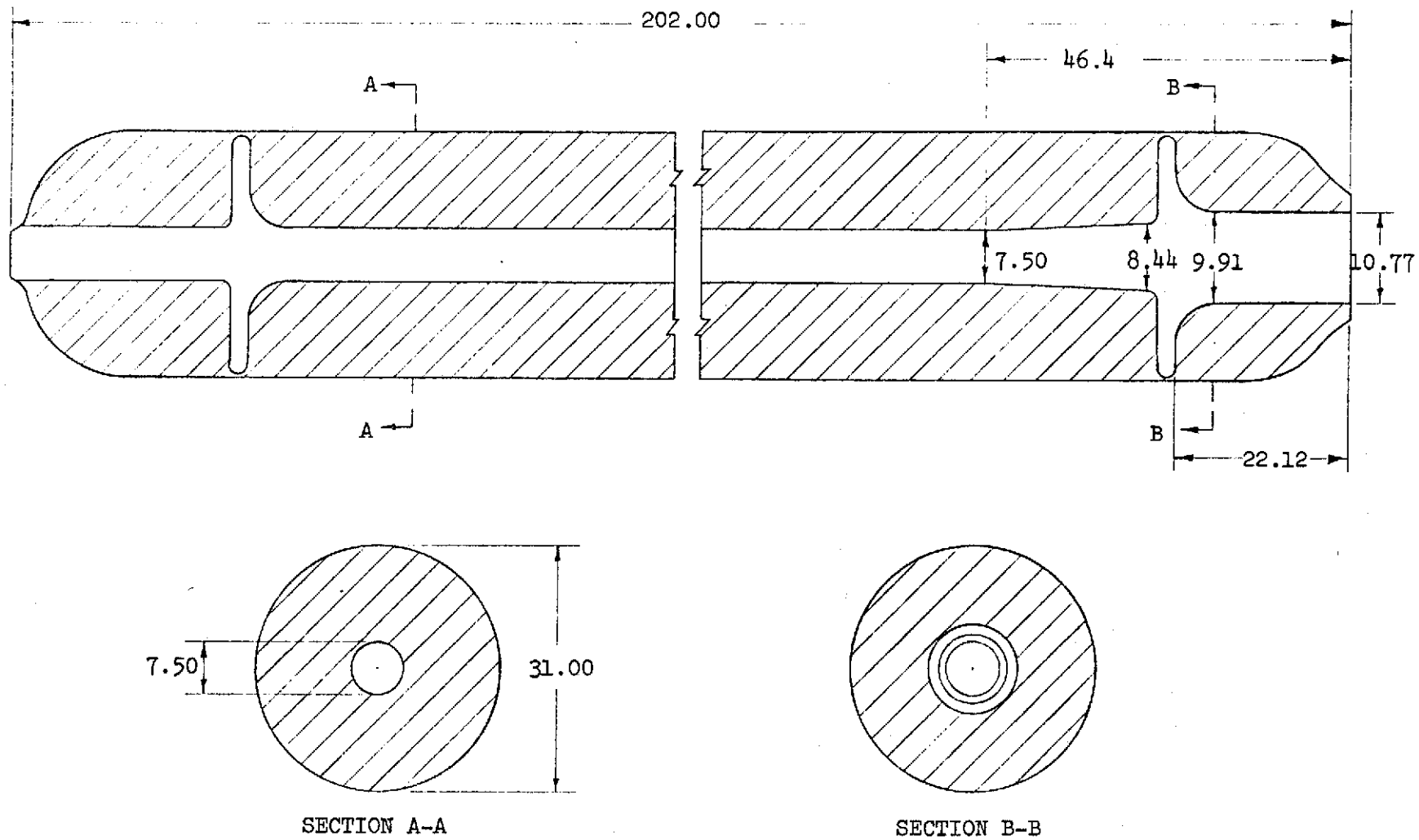


Figure V-3. Sketch of SRM Number 2 (Reference 9).
All dimensions are in inches.

TABLE V-4. INPUT DATA FOR SRM NO. 2

PROPELLANT CHARACTERISTICS					
RHC=	0.002010				
A=	0.0530				
N=	0.274			C.P. GRAIN GEOMETRY	
ALPHA=	7.7	DO=	30.720		
BETA=	220.0	DI=	7.500		
MU=	1.2600E-07	DELDI=	0.940		
CSTAR=	5.2110E 03	S=	3.		
		THETAG=	0.0		
		LCCI=	173.00		
		LGNI=	0.0		
BASIC MOTOR DIMENSIONS					
L=	200.	THETCN=	1.57075		
TAU=	11.61	THETCH=	0.75035		
VCI=	1.3000E 04				
VCF=	1.3000E 05			INERT WEIGHT INPUTS	
DE=	2.2950E 01	PIPK=	9.0000E-04		
DII=	8.2900E 00	DEMP=	2.3000E 01		
THETA=	1.7450E-01	SIGMAP=	3.0000E-02		
ALFAN=	2.2570E-01	SIGMAS=	0.0		
LTAP=	1.3000E 01	N1=	3.0000E 00		
XT=	6.0000E-01	N2=	0.0		
ZO=	0.0	SYCNOM=	1.5000E 05		
		DCC=	3.0900E 01		
		PSIC=	1.2000E 00		
BASIC PERFORMANCE CONSTANTS					
DELTAY=	0.050	DELC=	2.8300E-01		
XCUT=	9999.99	LCC=	1.7200E 02		
DPGUT=	9999.99	NSEG=	1.0000E 00		
ZETA=	0.970	HCN=	1.5000E 01		
TB=	40.0	SYNNOM=	3.5000E 04		
HB=	0.	PSIS=	1.4000E 00		
GAM=	1.160	PSIA=	2.0000E 00		
RADER=	0.0040	K1=	2.0800E-01		
		K2=	9.2500E-02		
		PSIINS=	2.0000E 00		
		DELINS=	5.2000E-02		
GRAIN CONFIGURATION					
INPUT=	3	KEH=	3.0000E-03		
GRAIN=	1	KEN=	1.2000E-02		
STAR=	0	DLINER=	3.3000E-02		
NT=	0.	TAUL=	6.5000E-02		
ORDER=	2	WA=	0.0		

TABULAR VALUES

Y	ABNK	Y	ABNK	Y	ABNK
0	1586	4.0	1290	8.0	747
1.0	1562	5.0	1224	9.0	397
2.0	1549	6.0	1089	9.72	0
3.0	1457	7.0	944	30.0	0

All other A's = 0

TABLE V-5. SAMPLE PRINTOUT OF TRANSIENT VALUES FOR SRM NO. 2

TIME=	36.9	Y=	11.500				
RNOZ=	3.0853E-01	RHEAD=	3.0863E-01	PON0Z=	6.1948E 02	PHEAD=	6.2027E 02
PTAR=	1.2804E 01	MNOZ=	4.6511E-02	SUMAB=	1.1023E 04	SG=	0.0
PATM=	1.4696E 01	CFVAC=	1.7184E 00	FVAC=	5.8179E 04	F=	5.2372E 04

TIME=	37.0	Y=	11.550				
RNOZ=	3.0743E-01	RHEAD=	3.0754E-01	PON0Z=	6.1150E 02	PHEAD=	6.1227E 02
PTAR=	1.2800E 01	MNOZ=	4.6525E-02	SUMAB=	1.0921E 04	SG=	0.0
PATM=	1.4696E 01	CFVAC=	1.7184E 00	FVAC=	5.7446E 04	F=	5.1639E 04

****TAIL OFF BEGINS****

TIME=	37.2	Y=	11.600				
RNCZ=	2.6402E-01	RHEAD=	2.6402E-01	PON0Z=	3.5084E 02	PHEAD=	3.5084E 02
PTAR=	1.2796E 01	MNOZ=	4.6540E-02	SUMAB=	7.3614E 03	SG=	0.0
PATM=	1.4696E 01	CFVAC=	1.7183E 00	FVAC=	3.2968E 04	F=	2.7160E 04

****BEGIN HALF SECOND TRACE****

TIME=	37.4	Y=	11.650				
RNOZ=	0.0	RHEAD=	0.0	PON0Z=	1.7542E 02	PHEAD=	1.7542E 02
PTAR=	1.2796E 01	MNOZ=	4.6540E-02	SUMAB=	0.0	SG=	0.0
PATM=	1.4696E 01	CFVAC=	1.7183E 00	FVAC=	1.6489E 04	F=	1.0681E 04

TIME=	37.9	Y=	11.650				
RNOZ=	0.0	RHEAD=	0.0	PON0Z=	5.7567E-01	PHEAD=	5.7567E-01
PTAR=	1.2796E 01	MNOZ=	4.6540E-02	SUMAB=	0.0	SG=	0.0
PATM=	1.4696E 01	CFVAC=	1.7184E 00	FVAC=	5.4115E 01	F=	0.0

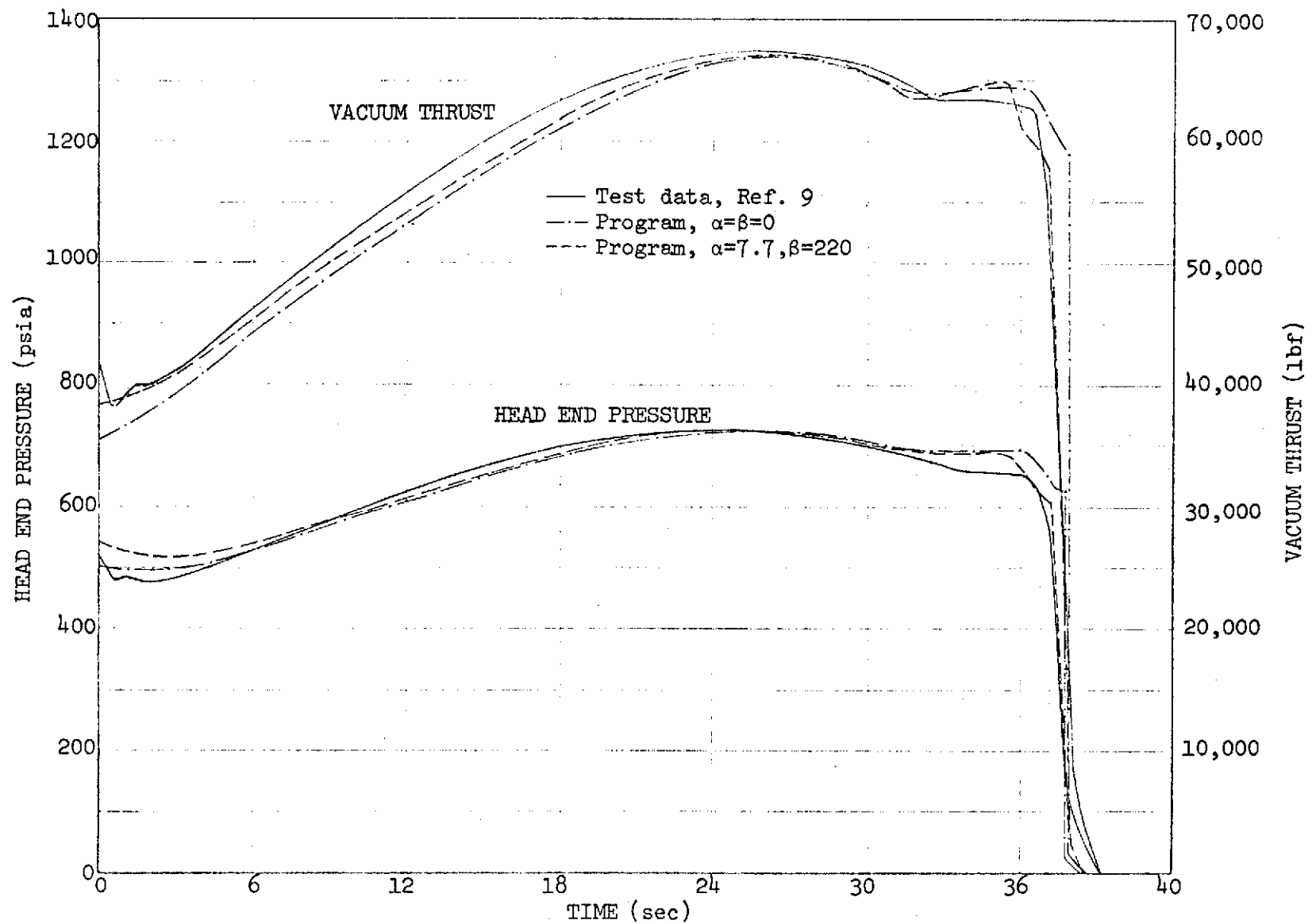


Figure V-4. Head end pressure and vacuum thrust versus time for SRM number 2.

TABLE V-6. COMPARISON OF COMPUTED WEIGHTS AND RELATED VALUES
WITH RESULTS OF OTHERS

Parameter	<u>SRM 1</u>		<u>SRM 2</u>	
	<u>Present Program</u>	<u>Reference 4</u>	<u>Present Program</u>	<u>Reference 9</u>
W_p , lbm	1,227,600	1,214,327	8204	8220
$P_{h \max}$, psia	999	—	728	721
I_{sp} , lbf-sec/lbm	262.2	—	235.6	237.6
$I_{sp \text{ vac}}$, lbf-sec/lbm	272.6	270.9	262.2	266.1
I_T , lbf-sec	321,830,000	—	1,932,600	1,953,400
$I_T \text{ vac}$, lbf-sec	334,650,000	331,300,000	2,151,200	2,187,000
P_{meop} , psia	1139	1,000	811	799
τ_c , in.	0.656	0.570	0.102	0.100
W_c , lbm	123,950	102,724	573	971
W_n , lbm	12,580	11,862	113	342
W_{ins} , lbm	9,340	11,906	101	143
W_ℓ , lbm	1,302	1,278	42	27
W_m , lbm*	1,374,800	1,342,097	9033	9703

* No allowance is made for W_a .

VI. CONCLUDING REMARKS

The SRM design and analysis program presented in this report is based in part on approximate equations and methods but gives a reasonable description of SRM performance. Computational times are reasonably short. The designer should be able to acquire quickly the skills necessary to prepare input data.

Additional comparisons should be made between program and actual test results to establish clearly the accuracy of the program.

Obvious improvements possible in the program fall into two categories: 1) several modifications may be made to reduce the number of input variables without compromising accuracy, and 2) the method of handling the end conditions can be made more accurate. Most, but not necessarily all, changes in the latter category are toward more sophisticated programs which it was the objective of this analysis to avoid.

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9. "Technical Description of the TX354-5 Rocket Motor," Huntsville Division, Thiokol Chemical Corporation, Control No. U-67-1005A, May 9, 1967.

APPENDIX

DESIGN DATA COMPILATION

Here the effects of changes in various design parameters are examined and the results compiled in the form of traces of nozzle end stagnation pressure versus time. The baseline SRM used in the analysis is the same as SRM 1 of Section V. Except as noted below, the effect of each design parameter is determined separately; that is, all other input parameters given in Table V-1 are kept the same while computations are made for different values of the parameter under consideration. Also, the parameters selected for variation have in general only minor influence on the propellant volume so that the designs analyzed all load approximately the same amounts of propellant. The stagnation pressure at the nozzle end of the grain was selected for plotting versus the burning time because this characteristic is most descriptive of SRM performance. For example, the vacuum thrust may be obtained by multiplying the stagnation pressure by the product of the vacuum thrust coefficient and throat cross-sectional area. This product is a constant at a given time for all the variations considered except in the case of changes in the radial erosion rate of the nozzle throat. Study of the resulting plots should provide a guide to the designer in predicting at least the qualitative effects of changes in the various design characteristics on the performance of SRMs of the general type considered.

Radial Erosion Rate

Figure A-1 shows the effect of the radial erosion rate ($E_n = \text{RADAR}$) on the stagnation pressure at the nozzle end of the grain at various times.

As can be seen, the effect is substantial during the latter portion of the trace, the higher erosion rates producing significant lower stagnation pressure as the burning progresses. The effect on thrust, however, will be much less throughout the trace because of the larger throat areas which accompany higher erosion rates. Of somewhat more significance is the influence of radial erosion rate on the burning time of the SRM. As seen from the plot, the burning time variation is approximately four percent over the range of erosion rates considered.

Initial Grain Temperature

This parameter was investigated by adjusting the burning rate coefficient ($a=A$) according to the relationship

$$a = a_o \exp[(\pi_p)_k (1-n)\Delta T_i]$$

where a_0 is the burning rate of the baseline design, which is based on an initial temperature of 70°F, and ΔT is the variation from this temperature. The results are shown on Figure A-2 for various initial grain temperatures. The generally higher pressures and reduced burning times at the higher initial grain temperatures are evident from the plot.

Number of Burning Slot Faces

The results are shown on Figure A-3. An increase in the number of burning slot faces ($S=S$) in the circular perforated grain section of the baseline design produces initially higher chamber pressures because of the added burning surface area, but the trace is more regressive because of the additional decrease in propellant length with distance burned. It is of interest that the net effect is an essentially constant burning time over the range of this parameter considered.

Number of Star Points

Influences of the number of truncated star points ($n_p = NP$) on the trace of nozzle end stagnation pressure versus time for a fixed length of the star grain in the baseline design is shown on Figure A-4. The results are qualitatively similar to those of the variations in the number of burning slot faces ($S=S$) in the circular perforated grain section.

Length of Star Grain

Influences of changes in the initial length of the star grain ($L_{Gsi}=L_{GSI}$) were evaluated for a fixed overall grain length ($L_{Gsi} + L_{Gci} = 1196.50$ inches). The results plotted in Figure A-5 confirmed the expected higher initial pressures and greater regressivities accompanying the greater lengths of star grain.

Taper

Effects of taper were investigated by independently varying the aft end taper and the taper of the main portion of the grain. The former variation involved changes in the difference in web thickness ($X_{Ta} = XT$) over the aft end tapered length and the latter involved changes in the differences in web thickness ($z_0 = ZO$) over the remaining portion of the grain. The results are shown in Figures A-6 and A-7 for the final portion of the traces which is the only part where a substantial effect is produced. The pressure changes for the initial portion of the traces were less than 1.7 percent over the range of variations considered, these changes being produced by changes in the parameter $\Delta D_i = DELDI$ corresponding to and equal to the changes in the parameter $z_0 = ZO$. It should be noted that the effects of z_0 variations are

coupled with the effects of an X_{Ta} value of 5.0 inches which is the value for the baseline design. Influences of the two types of changes are similar except that z_0 increases produce much greater regressivities and burning times. The longer burning times result from the burning rate reductions which accompany the reduced pressures. It is noteworthy that if each center grain segment were tapered identically with a z_0 over each segment equal to the z_0 corresponding to a continuous taper over the entire grain length (the present case) the results would be essentially the same, and therefore the alternate design could be treated by the simplified computer program.

Thrust Termination Passageways

Figure A-8 illustrates the effects produced by use of two thrust terminations in conjunction with SRM 1. The passageways were assigned the same diameter ($D_{TP} = DTP$) as the nozzle throat. The angle between the axis of the passage and the motor axis ($\theta_{TP} = \text{THETTP}$) was taken as 45° . The effective web thickness ($\tau_{Teff} = \text{TAUEFF}$) was assigned various values. The discontinuities in the resultant traces occurring at $y = \tau_{Teff}$ are the result of the method used to approximate the effects of the passage geometry in the simplified analysis and will not occur in actual firings. To obtain a better approximation, the traces should be smoothed out for some distance ahead of the discontinuity depending on the precise geometrical situation, i.e., the location of the axis of the passage with respect to the head end of the motor.

Erosive Burning

Erosive burning was investigated by arbitrary variation of the constants ($\alpha_{eb} = \text{ALPHA}$ and $\beta = \text{BETA}$) in the Robillard-Lenoir burning rate equation. The values $\alpha = 7.7$ and $\beta = 88.7$ are the approximate values given by Peretz (Reference 7) for a composite unmetallized 78% ammonium perchlorate - 22% polyester propellant. Computations were made for these values and for changes of 50% in each variable. The results are shown in Figure A-9 along with the baseline results.

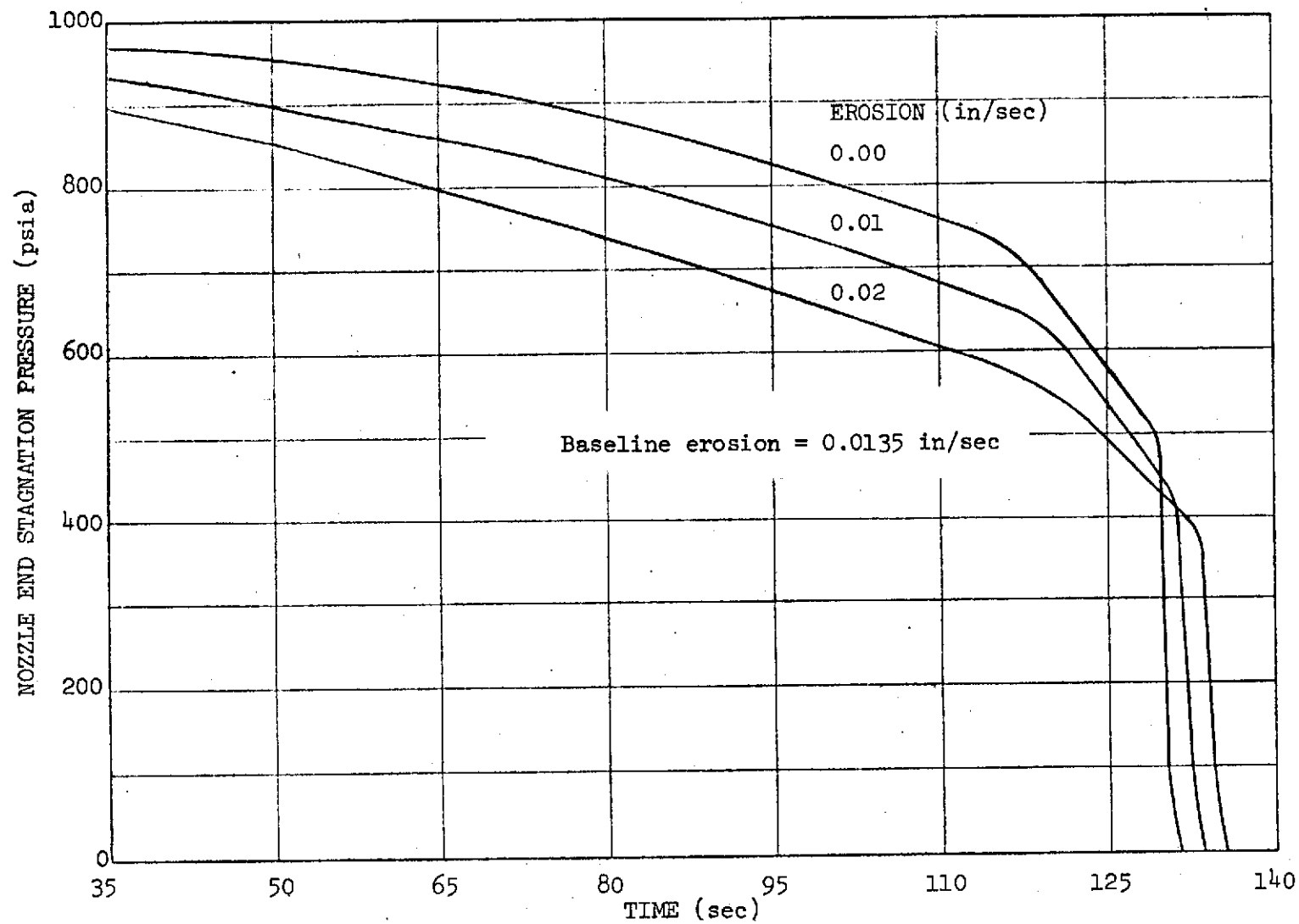


Figure A-1. Nozzle end stagnation pressure versus time for various nozzle radial erosion rates (SRM 1).

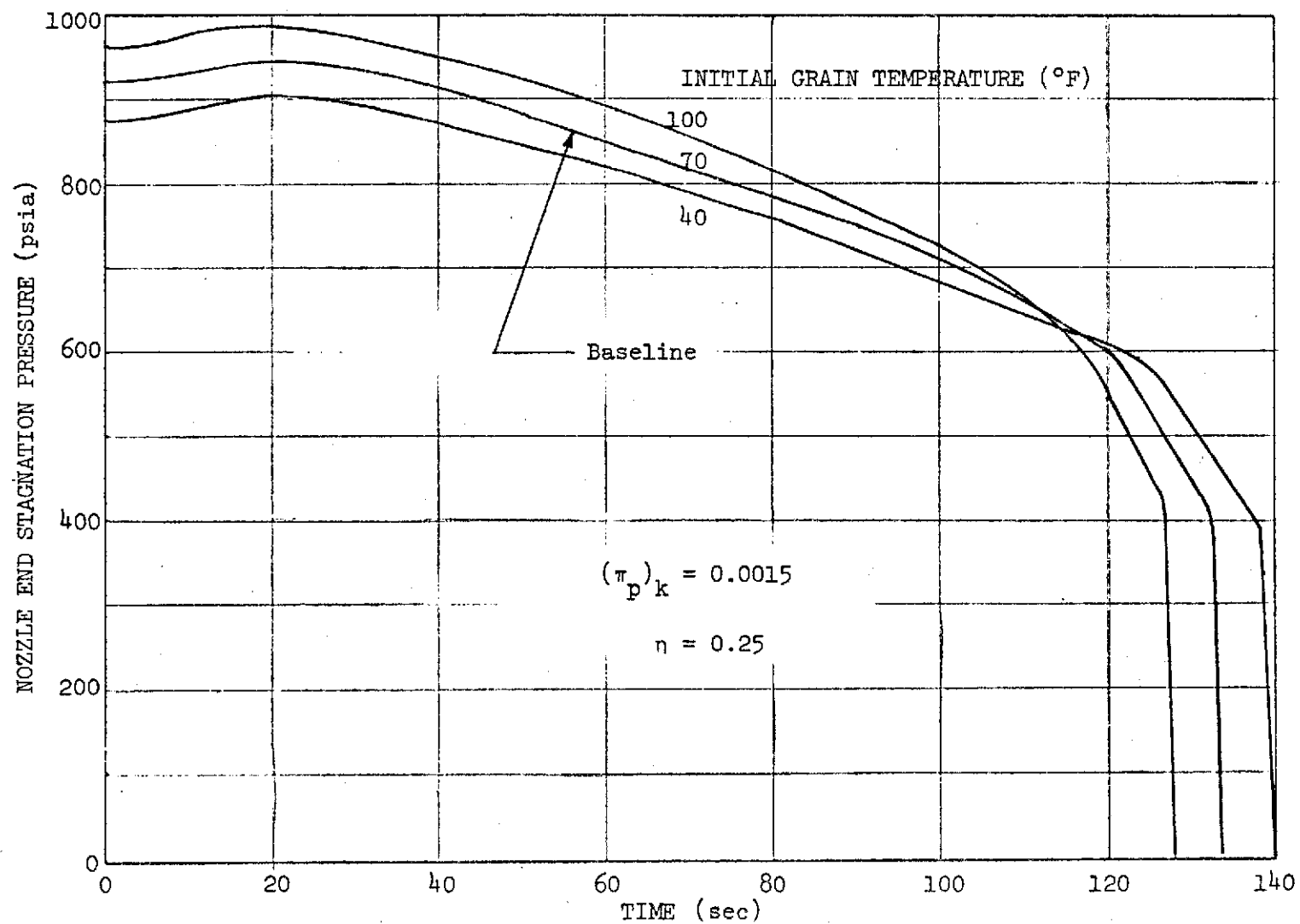


Figure A-2. Nozzle end stagnation pressure versus time for various initial grain temperatures (SRM 1).

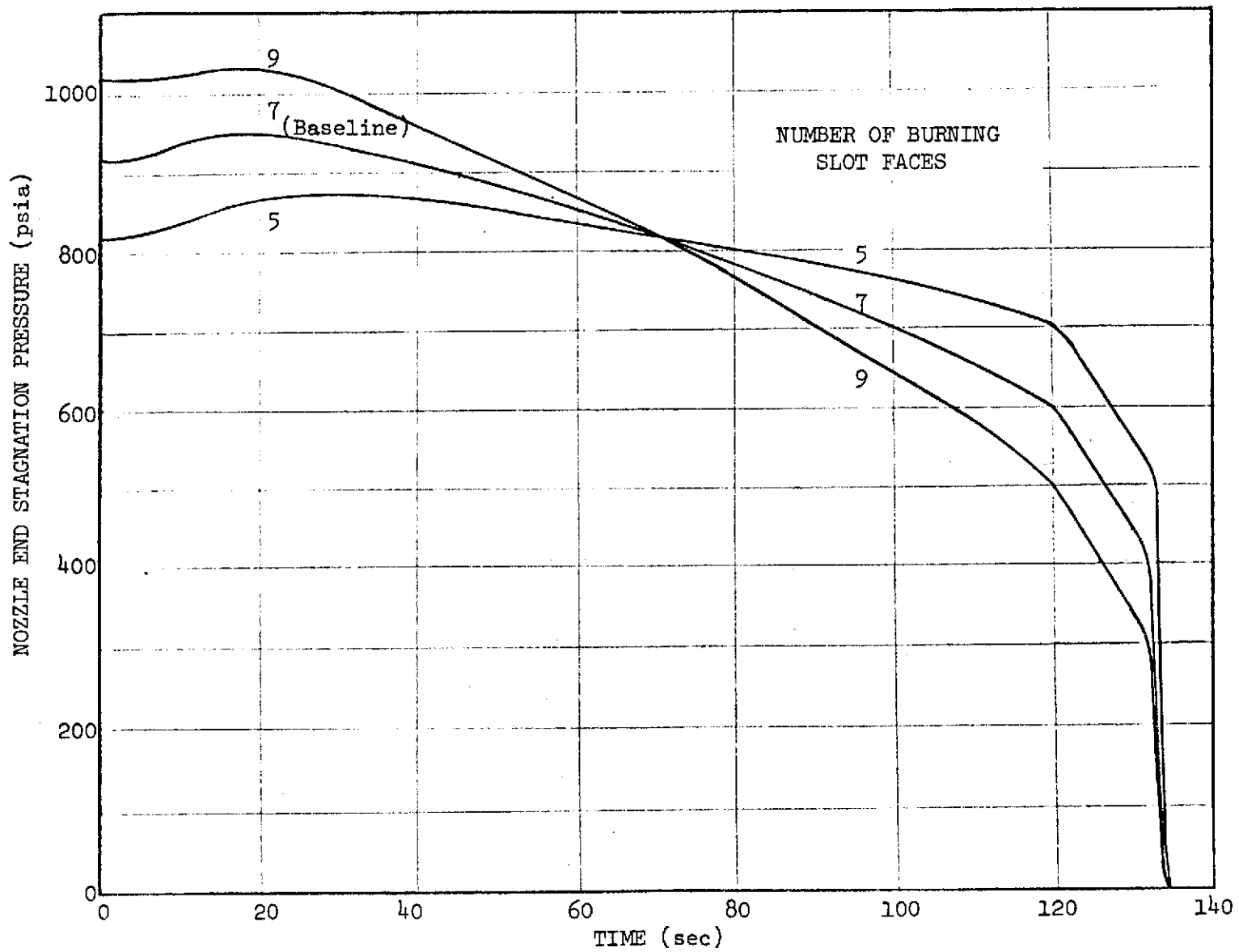


Figure A-3. Nozzle end stagnation pressure versus time for various numbers of burning slot faces (SRM 1).

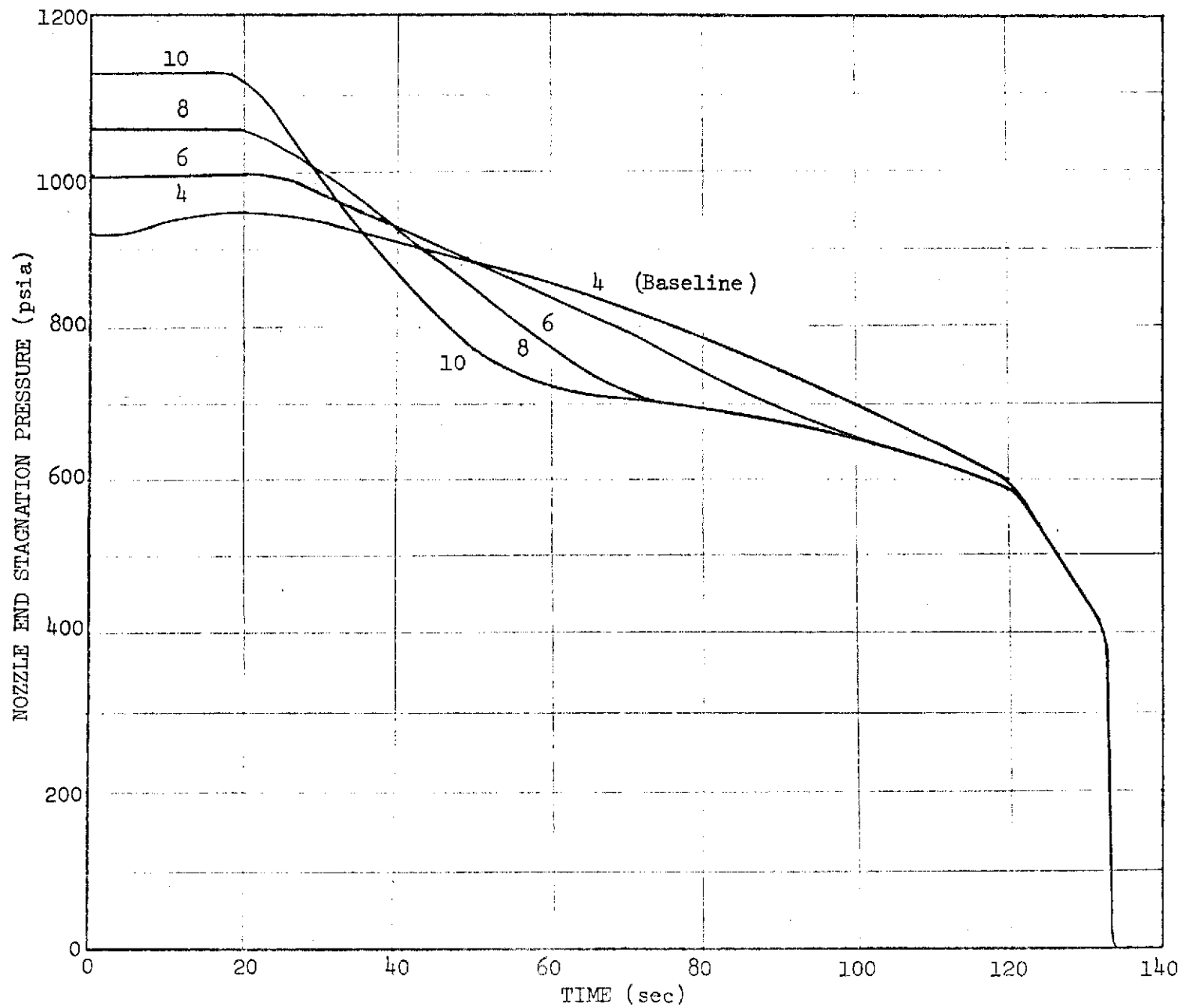


Figure A-4. Nozzle end stagnation pressure versus time for various numbers of star points (SRM 1)

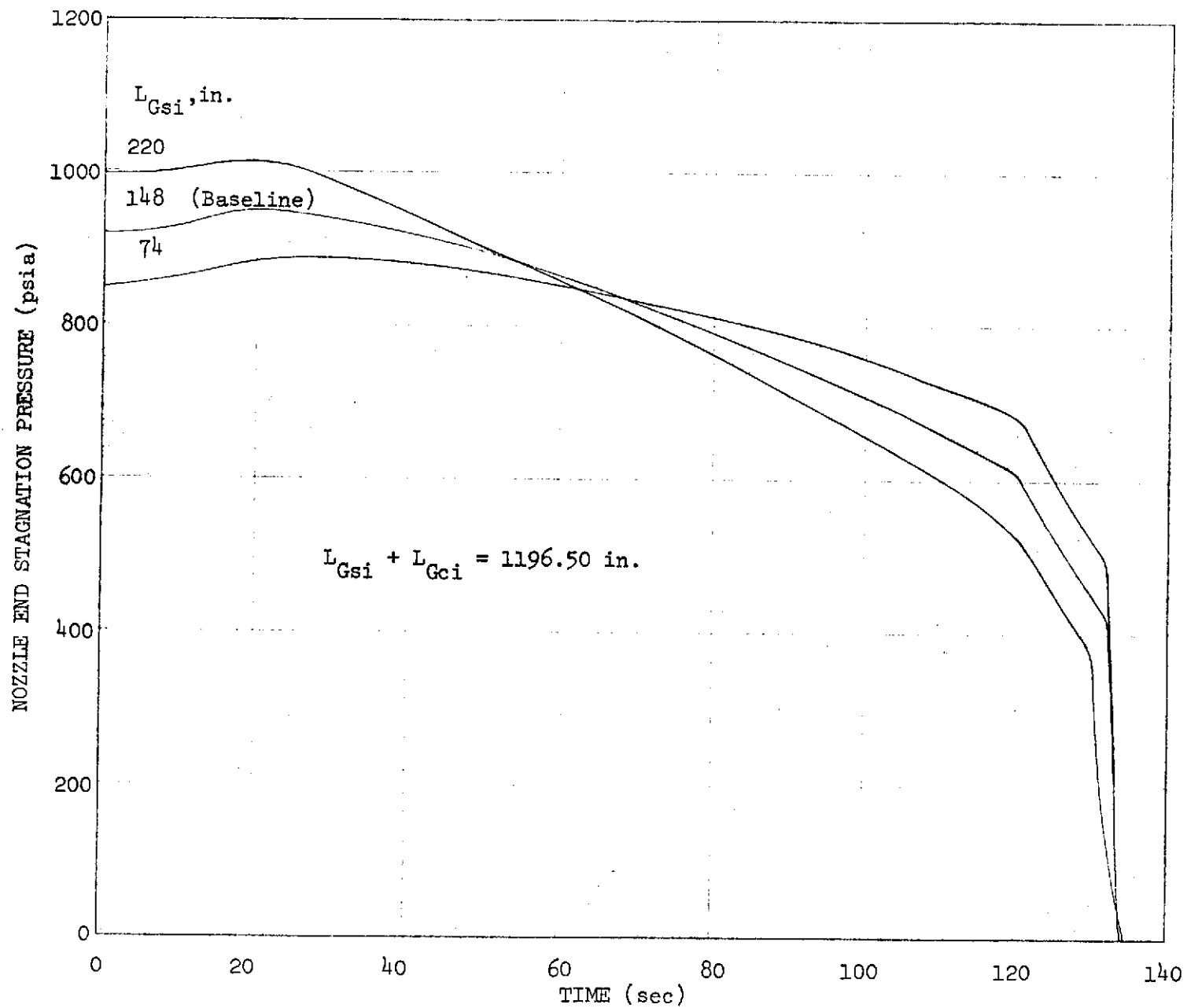


Figure A-5. Nozzle end stagnation pressure versus time for various lengths of star grain (SRM 1).

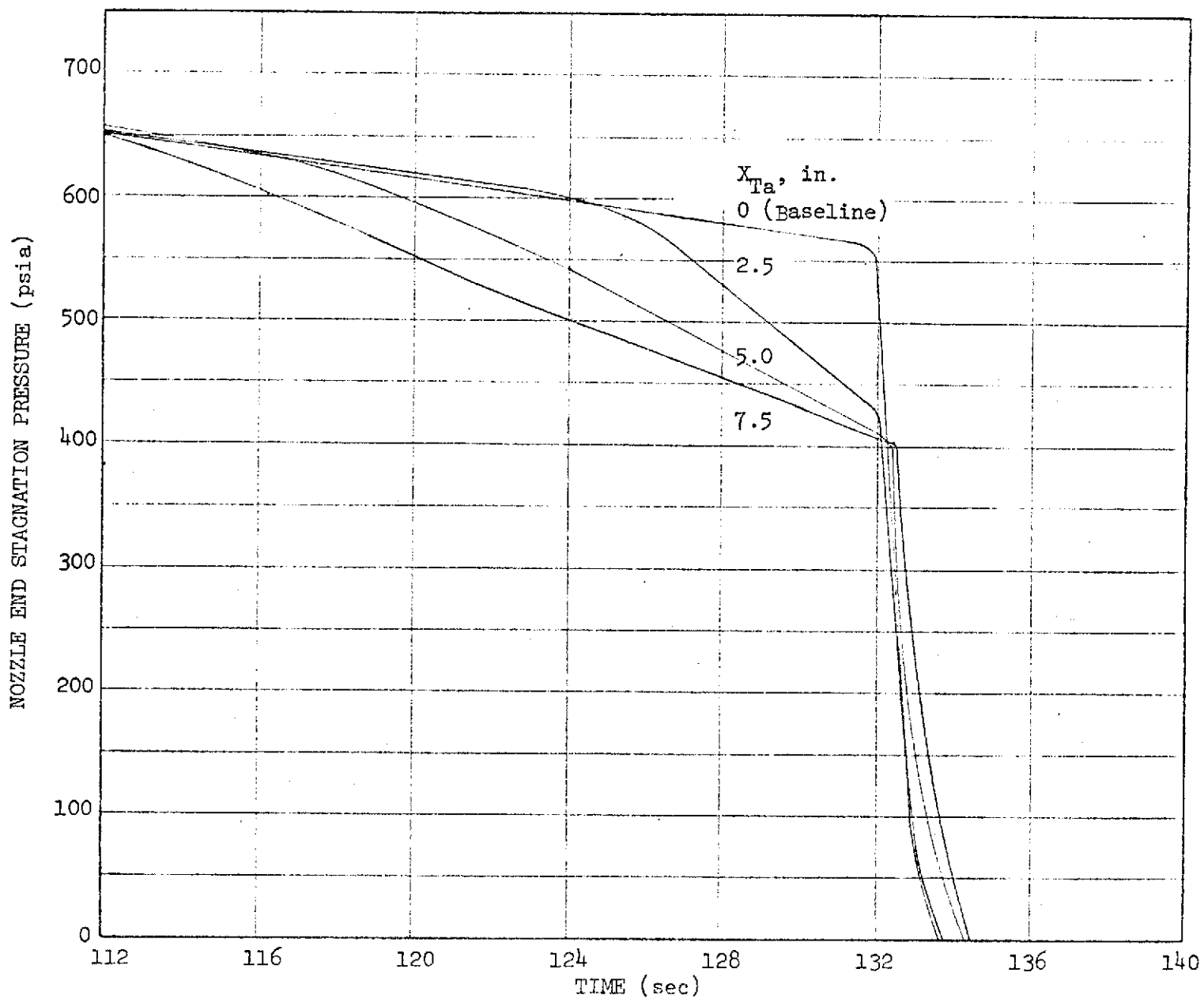


Figure A-6. Nozzle end stagnation pressure versus time for various differences in web thickness x_{Ta} over the aft end tapered grain length (SRM 1).

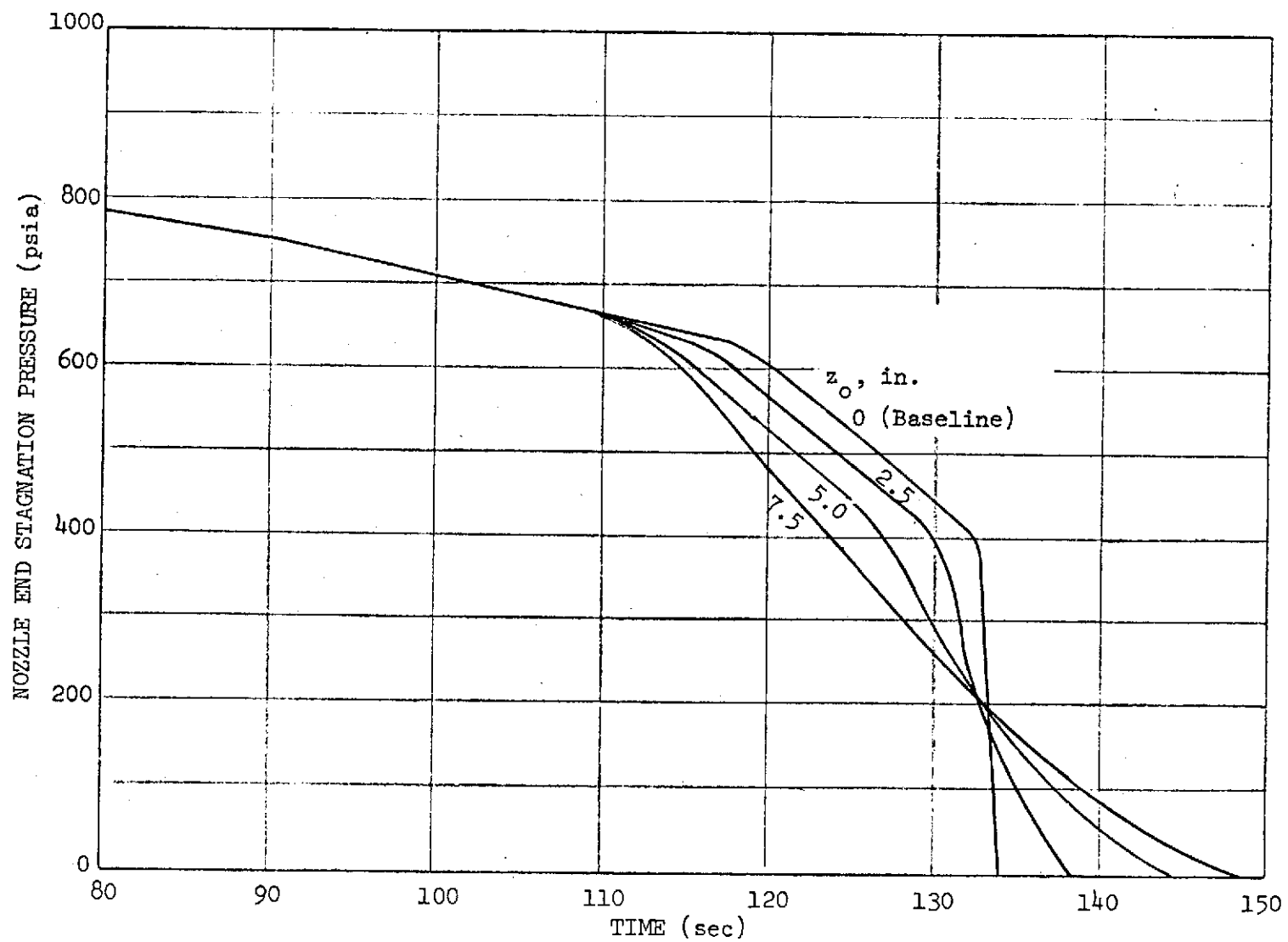


Figure A-7. Nozzle end stagnation pressure versus time for various differences in web thickness z_o between the ends of the ends of the main grain (SRM 1).

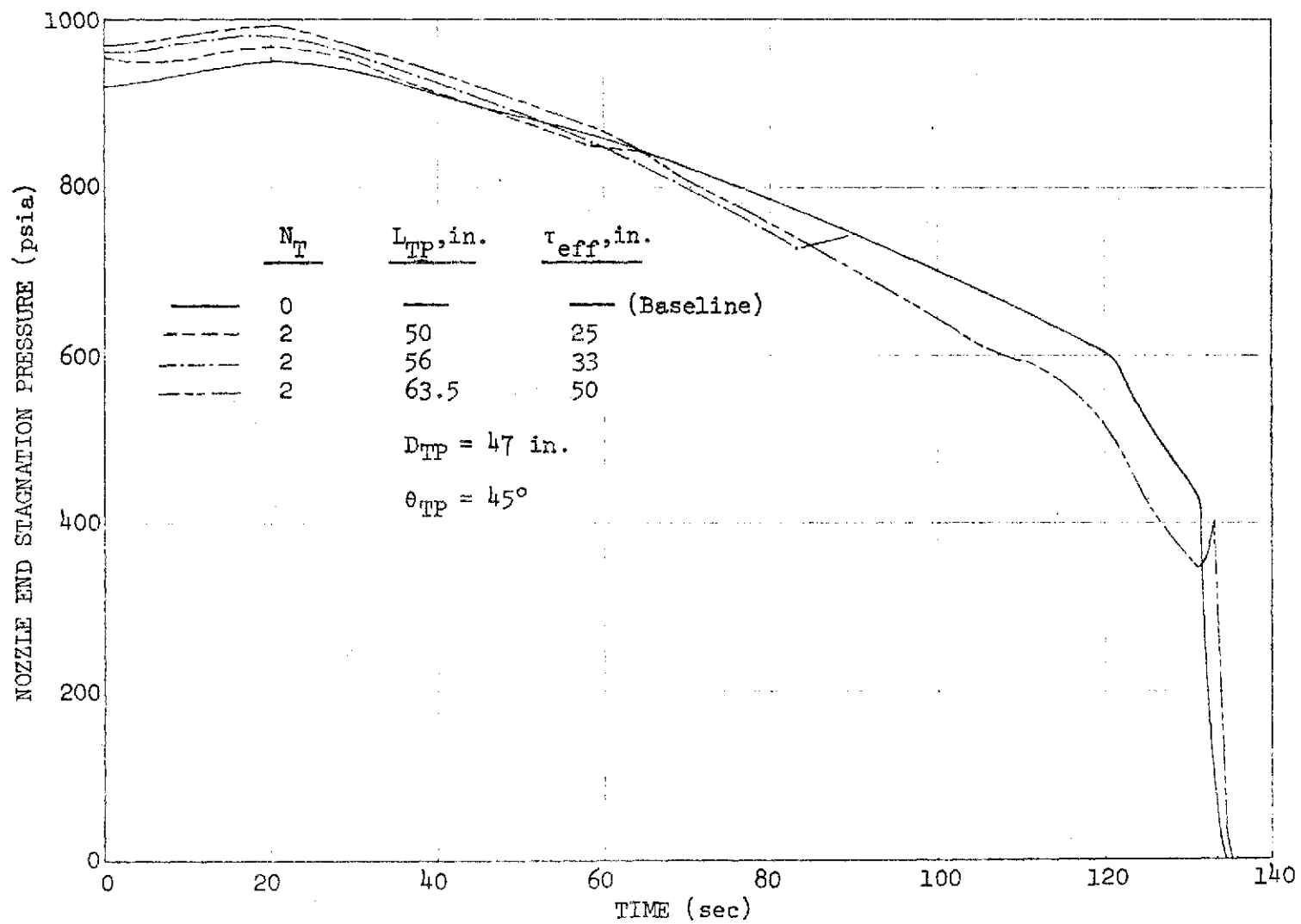


Figure A-8. Nozzle end stagnation pressure versus time with and without thrust termination ports (SRM 1).

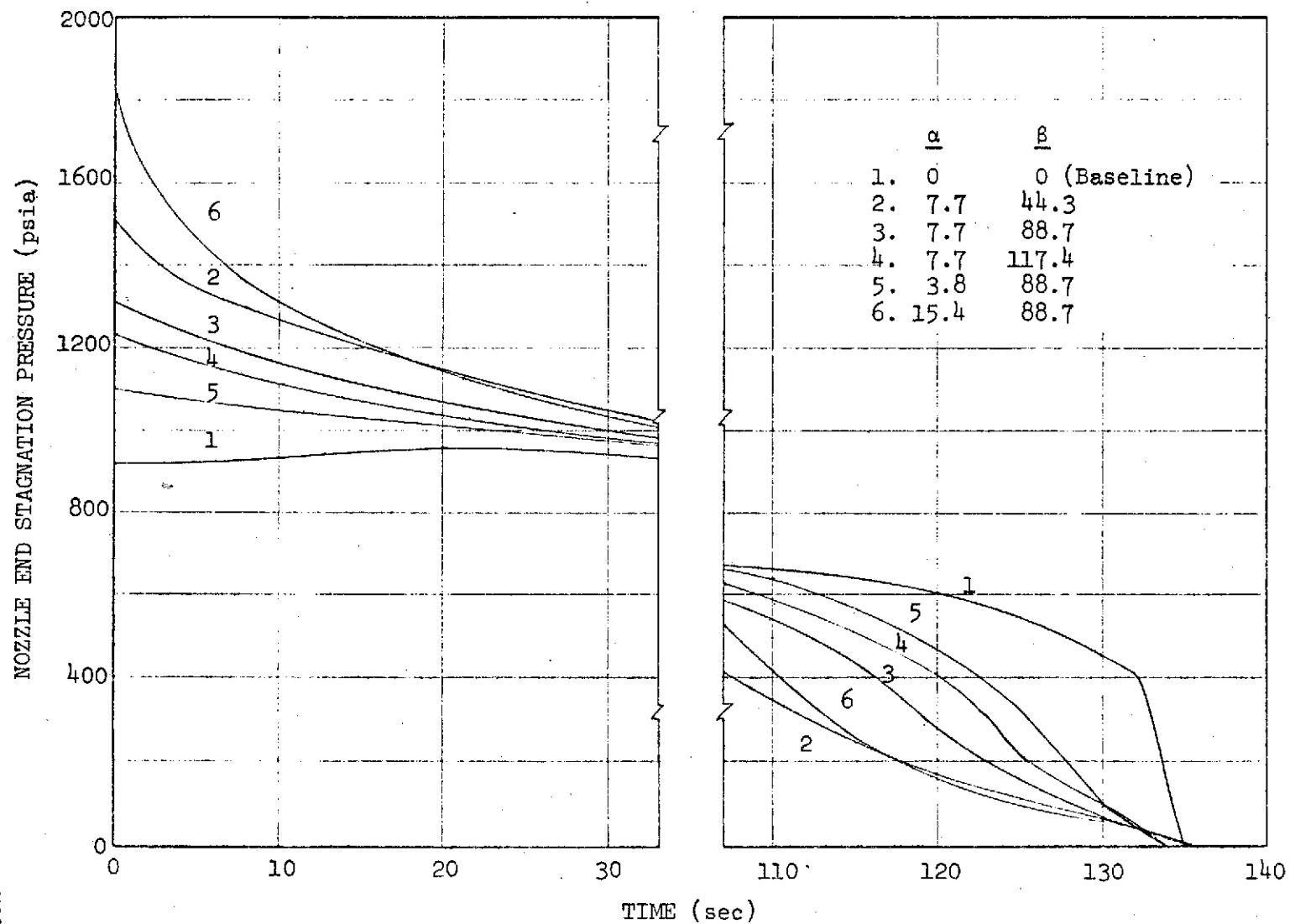


Figure A-9. Nozzle end stagnation pressure versus time for various values of the erosive burning constants (SRM 1)